Math 2001 - Assignment 4

Due September 23, 2015

(1) Are the following equalities true for all statements P, Q, R?
   (a) $P \Leftrightarrow Q \;=\; \sim P \Leftrightarrow \sim Q$
   (b) $(P \Rightarrow Q) \Rightarrow R \;=\; P \Rightarrow (Q \Rightarrow R)$

(2) Find a statement in P, Q and R that is true exactly for the following instances:

P	Q	R	
T	F	F	T
F	T	F	T

(3) [1, Section 2.7]: Exercises 4,6,8

(4) Formulate the following sentences using quantifiers and logical operations. Are they true?
   (a) For all integers n we have that $n(n+1)$ is even.
   (b) If x^2 is rational, then so is x.
   (c) There exists a real number z such that $x + z = x$ for every real x.
   (d) Every real number is smaller than some integer.

(5) Negate the following sentences:
   (a) $xy = 0$ iff $x = 0$ or $y = 0$
   (b) The derivative of a polynomial function f is 0 iff f is constant.
   (c) $\exists x \in \mathbb{R} : x^2 = -1$
   (d) $\forall r \in \mathbb{R} : \sin(r\pi) = 0 \Leftrightarrow r$ is an integer

(6) Are the following sentences true? Negate them:
   (a) There exists a right triangle that is not isosceles.
   (b) $\forall x \in \mathbb{R}^+ \; \exists n \in \mathbb{N} : \frac{1}{n} \leq x$.
   (c) $\exists m \in \mathbb{N} \; \forall p \in \mathbb{N} : p$ prime $\Rightarrow p \leq m$

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`