

Math 2001 - Assignment 3

Due September 16, 2015

- (1) Are the following statements? If so, determine whether they are true or false.
- (a) Some swans are black.
True
 - (b) Every real number is an even integer.
False
 - (c) If x is an even integer, then $x + 1$ is odd.
True
 - (d) $2x = 1$
Not a statement
 - (e) There is no largest natural number.
True
- (2) [1, Section 2.3]: Exercises 1,2,3,10
- 1. If the determinant of a matrix is not 0, then the matrix is invertible.**
 - 2. If a function is differentiable, then it's continuous.**
 - 3. If a function is integrable, then it's continuous.**
 - 10. If the discriminant is negative, then the quadratic equation has no real solution.**
- (3) Formulate the negations of the following statements. Are they true?
- (a) Yellowstone is in Colorado or my geography is wrong.
Version 1: It is not true that Yellowstone is in Colorado or my geography is wrong.
Version 2: Yellowstone is not in Colorado and my geography is right. True.
 - (b) 2 is even, and 3 is even.
It is not true that 2 is even, and 3 is even.
2 is odd or 3 is odd. True
 - (c) $2^n + 1$ is a prime number for every $n \in \mathbb{N}$.
 $2^n + 1$ is not a prime number for every $n \in \mathbb{N}$. True
 - (d) There exists an even prime.
There does not exist an even prime.
All primes are odd. False
 - (e) If the integer x is a multiple of 6, then x is even.
It is not true that if x is a multiple of 6, then x is even.
 x is a multiple of 6 and x is not even. False

- (4) Give truth tables for the following:
(a) $\sim (P \Rightarrow Q)$ (b) $P \wedge \sim P$ (c) $(\sim P) \Leftrightarrow Q$
- (5) Use truth tables to show that the following hold for all logical statements P, Q, R :
(a) $P \vee (P \wedge Q) = P$
(b) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$
- (6) For the game “poison” there is a pile of n stones. Two players alternate in taking one or two stones from it with player 1 starting. Whoever has to take the last stone (the poisoned one) loses.
(a) Describe the strategies for player 1 and for player 2 to win depending on the choice of n . Reason what the best move for each player is in any situation.
(b) What if the rules are changed so that each player is allowed to take 1,2, or 3 stones in a move? What is the strategy if each player takes between 1 and k stones?

REFERENCES

- [1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: <http://www.people.vcu.edu/~rhammack/BookOfProof/>