

# Math 2001 - Assignment 3

Due September 16, 2015

- (1) Are the following statements? If so, determine whether they are true or false.
  - (a) Some swans are black.
  - (b) Every real number is an even integer.
  - (c) If  $x$  is an even integer, then  $x + 1$  is odd.
  - (d)  $2x = 1$
  - (e) There is no largest natural number.
- (2) [1, Section 2.3]: Exercises 1,2,3,10
- (3) Formulate the negations of the following statements. Are they true?
  - (a) Yellowstone is in Colorado or my geography is wrong.
  - (b) 2 is even, and 3 is even.
  - (c)  $2^n + 1$  is a prime number for every  $n \in \mathbb{N}$ .
  - (d) There exists an even prime.
  - (e) If the integer  $x$  is a multiple of 6, then  $x$  is even.
- (4) Give truth tables for the following:
  - (a)  $\sim (P \Rightarrow Q)$
  - (b)  $P \wedge \sim P$
  - (c)  $(\sim P) \Leftrightarrow Q$
- (5) Use truth tables to show that the following hold for all logical statements  $P, Q, R$ :
  - (a)  $P \vee (P \wedge Q) = P$
  - (b)  $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$
- (6) For the game “poison” there is a pile of  $n$  stones. Two players alternate in taking one or two stones from it with player 1 starting. Whoever has to take the last stone (the poisoned one) loses.
  - (a) Describe the strategies for player 1 and for player 2 to win depending on the choice of  $n$ . Reason what the best move for each player is in any situation.
  - (b) What if the rules are changed so that each player is allowed to take 1,2, or 3 stones in a move? What is the strategy if each player takes between 1 and  $k$  stones?

## REFERENCES

- [1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: <http://www.people.vcu.edu/~rhammack/BookOfProof/>