Math 2001 - Assignment 2

Due September 9, 2015

(1) Describe the following using set builder notation:
   (a) $A =$ the set of points $(x, y) \in \mathbb{R}^2$ on the line through $(1, 2)$ and $(3, 4)$
   (b) $B =$ the set of points in $\mathbb{R}^2$ that lie on a circle with center $(0, 0)$ and radius 2

(2) For $U := \{x \in \mathbb{Z} : 1 \le x \le 8\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{x \in U : x$ is even $\}$, and $C = \{x \in U : x \ge 4\}$ compute:
   (a) $A \cap C_U(B)$ (b) $A \cup (B \cap C)$ (c) $(A - B) \cup B$

(3) Are the following true for all sets A, B in a universe U?
   (a) $A - B = B - A$
   (b) $A \cup B \subseteq (A \cap \bar{B}) \cup (B \cap \bar{A})$
   Either prove the equations or give an example where they fail.

(4) Show for all sets A, B in the universe U:

$$\overline{A \cup B} = \bar{A} \cap \bar{B} \qquad \text{(de Morgan's law)}$$

(5) Is the following true for all sets A, B, C?

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

Either prove it or give an example where the equation is false.

(6) Simplify and justify your answers:
   (a) $\bigcup_{n \in \mathbb{N}} (0, n]$ (b) $\bigcap_{n=1}^{3} \{nz : z \in \mathbb{Z}\}$ (c) $\bigcup_{A \in P(\mathbb{N})} A$