What I would STILL like to know about equational logic

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1970: McKenzie proved that the equational theory of any finite lattice is finitely axiomatizable.

1973: Kruse and L'vov proved that the equational theory of any finite ring is finitely axiomatizable.

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Why?

Where is the proof for the BOOK?

Baker proved more

Kirby Baker proved that every finite algebra of finite signature that generates a conguence distributive variety has a finitlely axiomatizable equational theory.

There is more: to any finite algebra satisfying Kirby's hypotheses, you can add finitely many new basic operations. The resulting finite algebra still has a finitely axiomatizable equational theory.

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Expandably Finitely Based Finite Algebras

Call a finite algebra of finite signature expandably finitely based provided every algebra obtained from it by adding finitely many additional basic operations has a finitely axiomatizable equational theories.

What do you think?

Are there others besides the ones generating a congruence distributive variety??

How about the 2 element algebras?

How about the paraprimal algebras?

How about some others?

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Most of the finite basis results, either explicitly or implicitly, use the condition that the variety generated by the finite algebra has a finite residual bound. Bjarni Jónsson speculated in the early 1970's that this condition might be sufficient.

Problem

Prove that any finite algebra of finite signature that supports a Taylor term must have a finitely axiomatizable equational theory.

The Question is Subtle

In 1995 Ralph McKenzie published a proof that there is no algorithm, which given a finite algebra of finite signature, would determine whether it's equational theory is finitely axiomatizable—settling Tarski's Finite Basis Problem.

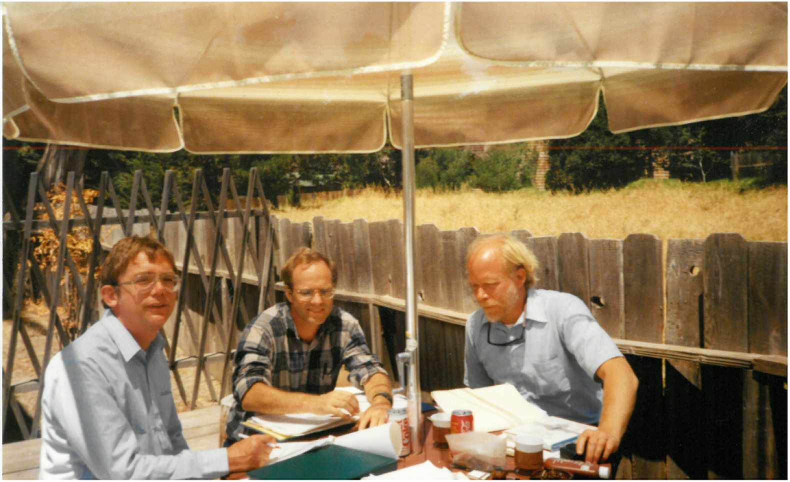
Finite Algebra Membership Problem

Let $\mathcal{V}$ be the variety generated by the finite algebra $\mathbf{A}$ of finite signature. The Finite Algebra Membership Problem for $\mathcal{V}$ is given a finite algebra $\mathbf{B}$ to determine whether $\mathbf{B} \in \mathcal{V}$.

Another Problem

Is there an algorithm, which given a finite algebra $\mathbf{A}$ of finite signature will determine whether the finite algebra membership problem for the variety generated by $\mathbf{A}$ can be settled in polynomial time?

One Last Problem



Do you believe in Volume II?