Math 3130 - Assignment 13

Due April 22, 2016

(109) [1, Section 6.1] Give 3 vectors of length 1 in $\mathbb{R}^3$ that are orthogonal to $\mathbf{u} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$.

(110) [1, Section 6.2] Which of the following are orthonormal sets?

$$A = \{\begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}, \begin{bmatrix} 0.8 \\ -0.6 \end{bmatrix}\}, \qquad B = \{\frac{1}{3}\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{18}}\begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}\}$$

(111) [1, Section 6.2] Let W be the subspace of $\mathbb{R}^3$ with orthormal basis $B = (\frac{1}{3}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix})$.

Compute the coordinates $[\mathbf{x}]_B$ for $\mathbf{x} = \begin{bmatrix} 7 \\ 4 \\ 4 \end{bmatrix}$.

(112) [1, 14, Section 6.2] Write $\mathbf{x} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ as a sum of a vector in $\text{Span}\{\mathbf{u}\}$ and a vector in $\text{Span}\{\mathbf{u}\}^\perp$ for $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$.

(113) [1, 6, Section 6.3] Check that $\mathbf{u}_1 = \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ form an orthogonal set and compute the orthogonal projection of $\mathbf{x} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$ onto $\text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

What is the distance from $\mathbf{x}$ to $\text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$?

(114) [1, 8, Section 6.3] Find the closest point to $\mathbf{x}$ in $\text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$:

$$\mathbf{x} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

(115) [1, 2, Section 6.4] Use the Gram-Schmidt algorithm to find orthonormal bases for the following subspaces:

$$U = \text{Span}\{\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}\}, \qquad W = \text{Span}\{\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix}\}$$

(116) [1, Section 6.4] Use the Gram-Schmidt process to transform the vectors in an orthonormal set.

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} -1 \\ 1 \\ 3 \\ -3 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(117) [1, Section 6.2–6.4] True or false. Explain your answers.

(a) Every orthogonal set is also orthonormal.
(b) Not every orthonormal set in $\mathbb{R}^n$ is linearly independent.
(c) For each $\mathbf{x}$ and each subspace W, the vector $\mathbf{x} - \text{proj}_W(\mathbf{x})$ is orthogonal to W.
(d) If a vector is both in a subspace W and in $W^\perp$, then it must be the zero vector.
(e) Multiplying the vectors in an orthogonal basis by non-zero scalars yields again an orthogonal basis.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.