Math 3130 - Assignment 11

Due April 8, 2016

(91) [1, Section 5.1] Are the following eigenvalues for the respective matrices? If so, give a basis for the corresponding eigenspace.

(a) $A = \begin{bmatrix} -4 & 1 & 1 \\ 2 & -3 & 2 \\ 3 & 3 & -2 \end{bmatrix}, \lambda = -5$ (b) $B = \begin{bmatrix} 3 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix}, \mu = 2$

(92) [1, Section 5.1] Give all eigenvalues and bases for eigenspaces. Do you need the characteristic polynomials?

(a) $A = \begin{bmatrix} -3 & 1 \\ 0 & -3 \end{bmatrix}$ (b) $B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

(93) [1, Section 5.2] Give the characteristic polynomial, all eigenvalues and bases for eigenspaces for $C = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$.

(94) [1, Section 5.2] Compute eigenvalues and eigenvectors for $D = \begin{bmatrix} -1 & 4 & 1 \\ 6 & 9 & 2 \\ 0 & 0 & -3 \end{bmatrix}$.

(95) [1, Section 5.1-5.2] Are the following true or false? Why?
  (a) The eigenvalues of a matrix are its diagonal entries.
  (b) A matrix is invertible iff 0 is not an eigenvalue.
  (c) Any two matrices with the same characteristic polynomial have the same eigenvalues.
  (d) Any two matrices with the same characteristic polynomial have the same eigenvectors.
  (e) If two matrices have the same eigenvalues, then they are equal.

(96) [1, Section 5.3] Are the matrices A, B, C, D in 92, 93, 94 diagonalizable? How?

(97) [1, Section 5.3] Let A be an $n \times n$-matrix. Are the following true or false? Why?
  (a) If A has n eigenvectors, then A is diagonalizable.
  (b) If a 4×4-matrix A has two eigenvalues with eigenspaces of dimension 3 and 1, respectively, then A is diagonalizable.
  (c) If A is invertible, then A is diagonalizable.
  (d) A is diagonalizable iff A has n eigenvalues (counting multiplicities).
  (e) If $\mathbb{R}^n$ has a basis of eigenvectors of A, then A is diagonalizable.

(98) [1, Section 5.3] Let A be the standard matrix for the reflection T of $\mathbb{R}^2$ on some line g throught the origin. What are the eigenvalues and eigenvectors of A? Can A be diagonalized? Hint: Consider what a reflection does to specific vectors.

(99) [1, Section 5.2] Let A be an $n \times n$-matrix with n eigenvalues $\lambda_1, \dots, \lambda_n$ (repeated according to their multiplicities). Show that

$$\det A = \lambda_1 \cdot \lambda_2 \cdots \lambda_n$$

Hint: Consider the characteristic polynomial $\det(A - \lambda I) = (\lambda_1 - \lambda) \cdots (\lambda_n - \lambda)$.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.