Math 3130 - Assignment 10

Due April 1, 2016
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(81) [1, Section 4.3] Let A be an $n \times n$ matrix. Is

$$H = \{\mathbf{x} \in \mathbb{R}^n \mid A\mathbf{x} = 2\mathbf{x}\}$$

a subspace of $\mathbb{R}^n$? Which conditions for a subspace are fulfilled by H?

(82) [1, Section 4.3] Let $\mathbf{u}, \mathbf{v}$ be linearly independent vectors in a vector space V.

(a) Find all $x_1, x_2 \in \mathbb{R}$ such that

$$x_1(\mathbf{u} + \mathbf{v}) + x_2(\mathbf{u} - \mathbf{v}) = 0.$$

(b) Are the vectors $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ linearly independent?

(83) [1, Section 4.4] Let $B = (\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3) = (1 + t, 1 + t^2, t + t^2)$ be a basis of $\mathbb{P}_2$, and let $\mathbf{u} = 1 + t^2$ and $\mathbf{v} = 2t$.

(a) Write both $\mathbf{u}$ and $\mathbf{v}$ as linear combination of $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$.

(b) Give the B-coordinates $[\mathbf{u}]_B$ and $[\mathbf{v}]_B$.

(84) For which $\lambda \in \mathbb{R}$ is

$$\lambda(\lambda^2 - 2)(\lambda^2 + 1)(\lambda^2 - 3\lambda + 2) = 0? \tag{1}$$

(85) [1, Section 3.2] For which $\mu \in \mathbb{R}$ has the matrix

$$B = \begin{bmatrix} 6 - \mu & 2 \\ -6 & -1 - \mu \end{bmatrix}$$

a determinant $\det B = 0$?

(86) [1, Section 4.2] Let

$$A = \begin{bmatrix} 6 & 2 \\ -6 & -1 \end{bmatrix}.$$

(a) Compute the matrices $A - 2I$, $A - 3I$, and $A - I$.

(b) Find all $\mathbf{x} \in \mathbb{R}^2$ such that $A\mathbf{x} = 2\mathbf{x}$. Give the parametric vector form for the solution set.
Hint: $A\mathbf{x} = 2\mathbf{x}$ iff $A\mathbf{x} = 2I\mathbf{x}$ iff $(A - 2I)\mathbf{x} = \mathbf{0}$.

(c) Find all $\mathbf{x} \in \mathbb{R}^2$ such that $A\mathbf{x} = 3\mathbf{x}$. Give the parametric vector form.

(d) Find all $\mathbf{x} \in \mathbb{R}^2$ such that $A\mathbf{x} = \mathbf{x}$. Give the parametric vector form.

(87) [1, Section 3.2] For which $\lambda \in \mathbb{R}$ has the matrix

$$B = \begin{bmatrix} -2 - \lambda & 0 & 2 \\ 6 & 2 - \lambda & -3 \\ -6 & 0 & 5 - \lambda \end{bmatrix}$$

a determinant $\det B = 0$?

(88) [1, Section 4.2] Let

$$A = \begin{bmatrix} -2 & 0 & 2 \\ 6 & 2 & -3 \\ -6 & 0 & 5 \end{bmatrix}$$

(a) Compute the matrices $A - 2I$ and $A - I$.

(b) Find all $\mathbf{x} \in \mathbb{R}^3$ such that $A\mathbf{x} = 2\mathbf{x}$. Give the parametric vector form for the solution set.

(c) Find all $\mathbf{x} \in \mathbb{R}^3$ such that $A\mathbf{x} = \mathbf{x}$. Give the parametric vector form.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.