Math 3130 - Assignment 9

Due March 18, 2016
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(73) [1, Section 4.2] Let $T\colon \mathbb{P}_3 \to \mathbb{R}, p \mapsto p(3)$, be the map that evaluates a polynomial p at $x = 3$.
  (a) Show that T is linear.
  (b) Determine the kernel and the range of T.
  (c) Is T injective, surjective, bijective?

(74) [1, Section 4.4]
  (a) Let $B = \left(\begin{bmatrix}1\\1\\3\end{bmatrix}, \begin{bmatrix}2\\-2\\1\end{bmatrix}\right)$ be a basis of a subspace H of $\mathbb{R}^3$. Compute the coordinates $[u]_B$ for $u = \begin{bmatrix}-5\\11\\5\end{bmatrix}$ in H.
  (b) Let $C = (1+t, t+t^2, 1+t^2)$ be a basis for $\mathbb{P}_2$. Compute the coordinates $[p]_C$ for $p = 2+t^2$.

(75) [1, Section 4.6]
  (a) If A is a 3×4-matrix, what is the largest possible rank of A? What is the smallest possible dimension of Nul A?
  (b) If the nullspace of a 4×6-matrix B has dimension 3, what is the dimension of the row space of B?

(76) [1, Sections 4.3-4.6] True or false? Explain your answers:
  (a) Any plane in $\mathbb{R}^3$ is isomorphic to $\mathbb{R}^2$.
  (b) A basis for V is a linear independent set that is as large as possible.
  (c) If $v_1, \dots, v_k$ are linearly independent in V, then $k \le \dim V$.
  (d) If B is an echelon form of A, then the pivot columns of B are a basis for Col A.
  (e) The row space of A^T is equal to the column space of A.

(77) [1, Section 3.1] Compute the determinant of the matrices by cofactor expansion. Pick a row or column that yields the least amount of computation:

$$A = \begin{bmatrix}0 & 1 & -3\\5 & 4 & -4\\0 & -3 & -4\end{bmatrix} \qquad B = \begin{bmatrix}1 & 0 & -3 & 0\\3 & 1 & 5 & 1\\2 & 0 & 0 & 0\\7 & 1 & -2 & 5\end{bmatrix}.$$

(78) [1, Section 3.1] **Rule of Sarrus for the determinant of 3×3-matrices.** Let

$$A = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{bmatrix}$$

Prove that

$$\det A = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - (a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33})$$

Hint: Expand $\det A$ across the first row.

(79) [1, Section 3.1] Give two $3{\times}3$-matrices with determinat 5. (Hint: triangular matrices.)

(80) [1, Section 3.2] Compute the determinants by row reduction to echelon form:

$$A = \begin{bmatrix} 3 & 3 & -3 \\ 3 & 4 & -4 \\ 2 & -3 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix}$$

(81) [1, Section 3.2] Consider $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(a) How does switching the rows effect the determinant? Compare $\det A$ and $\det \begin{bmatrix} c & d \\ a & b \end{bmatrix}$.

(b) How does adding a multiple of one row to the other row effect the determinant? Compare $\det A$ and $\det \begin{bmatrix} a & b \\ c+ra & d+rb \end{bmatrix}$.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.