Math 3130 - Assignment 8

Due March 11, 2016
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(64) [1, Section 4.2] Let U, V, W be vector spaces, and let $f\colon U \to V$ and $g\colon V \to W$ be linear mappings.
   (a) Show that the composition mapping $h\colon U \to W$, $\mathbf{u} \mapsto g(f(\mathbf{u}))$ is linear.
   (b) Does it make sense to ask whether $k\colon V \to V$, $\mathbf{u} \mapsto f(g(\mathbf{v}))$ is linear?

(65) [1, Section 4.2] Let U, V be vector spaces and $T\colon U \to V$ be a linear mapping. Show that $T(\mathbf{0}) = \mathbf{0}$.
   Hint: Write down $T(\mathbf{0}+\mathbf{0})$ in two different ways.

(66) [1, Section 4.2] Let U, V be vector spaces and $T\colon U \to V$ be a linear mapping. Show that the range $\operatorname{Rg} T$ is a subspace of V.

(67) [1, Section 4.3] Let $V = \{f\colon \mathbb{R} \to \mathbb{R}\}$ be a vector space of functions. Is the set $\{\cos t, \sin t, \sin(t + \frac{\pi}{4})\}$ linearly independent?
   Hint: Use the formula for $\sin(\alpha + \beta)$.

(68) [1, Section 4.4] Let $B = (\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix})$ and $C = (\begin{bmatrix}2\\5\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix})$ be bases of $\mathbb{R}^2$.
   (a) Find the standard matrix for $f\colon \mathbb{R}^2 \to \mathbb{R}^2$, $[\mathbf{u}]_B \mapsto \mathbf{u}$.
   (b) Find the standard matrix for $g\colon \mathbb{R}^2 \to \mathbb{R}^2$, $\mathbf{u} \mapsto [\mathbf{u}]_C$.
   (c) Find the standard matrix for $h\colon \mathbb{R}^2 \to \mathbb{R}^2$, $[\mathbf{u}]_B \mapsto [\mathbf{u}]_C$. Hint: $h(\mathbf{x}) = g(f(\mathbf{x}))$.

(69) [1, Sections 4.1–4.4] Let $B = (t, 2+t, t^2)$ and $C = (1, t+t^2, t^2)$ be bases of $\mathbb{P}_2$. Let h be the linear mapping $h\colon \mathbb{R}^3 \to \mathbb{R}^3$, $[\mathbf{u}]_B \mapsto [\mathbf{u}]_C$.
   (a) Compute $h(\mathbf{e}_1), h(\mathbf{e}_2), h(\mathbf{e}_3)$. Hint: If $[\mathbf{u}]_B = \mathbf{e}_1$, then $[\mathbf{u}]_C = h(\mathbf{e}_1) = ?$
   (b) Give the standard matrix of h.
   (c) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{P}_2$ such that $[\mathbf{v}_1]_B = \begin{bmatrix}1\\1\\1\end{bmatrix}$, $[\mathbf{v}_2]_B = \begin{bmatrix}2\\-1\\0\end{bmatrix}$, and $[\mathbf{v}_3]_B = \begin{bmatrix}0\\1\\2\end{bmatrix}$. Find $[\mathbf{v}_1]_C$, $[\mathbf{v}_2]_C$, $[\mathbf{v}_3]_C$.

(70) [1, Section 4.3] Let $\mathbf{b}_1 = \begin{bmatrix}1\\2\\-1\end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix}1\\1\\3\end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix}1\\2.5\\-5\end{bmatrix}$.
   (a) Find vectors $\mathbf{u}_1, \dots, \mathbf{u}_k$ such that $(\mathbf{b}_1, \mathbf{b}_2, \mathbf{u}_1, \dots, \mathbf{u}_k)$ is a basis for $\mathbb{R}^3$.
   (b) Find vectors $\mathbf{v}_1, \dots, \mathbf{v}_\ell$ such that $(\mathbf{b}_3, \mathbf{v}_1, \dots, \mathbf{v}_\ell)$ is a basis for $\mathbb{R}^3$.
   Prove that your choices for (a) and (b) form a basis.

(71) [1, Sections 4.3, 4.5] Let

$$A = \begin{bmatrix} -5 & 8 & 0 & -17 & -2 \\ 3 & -5 & 1 & 5 & 1 \\ 11 & -19 & 7 & 1 & 3 \\ 7 & -13 & 5 & -3 & 1 \end{bmatrix}.$$

   Find bases and dimensions for $\operatorname{Nul} A$, $\operatorname{Col} A$, and $\operatorname{Row} A$, respectively.

(72) [1, Section 4.5] A 177×35 matrix A has 19 pivots. Find $\dim \operatorname{Nul} A$, $\dim \operatorname{Col} A$, $\dim \operatorname{Row} A$, and $\operatorname{rank} A$.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.