Math 3130 - Assignment 5

Due February 19, 2016
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Please write problems (37), (38), (39) on a sheet of paper separate from the rest.

(37) Prove the following part of the Invertible Matrix Theorem: Let A be an $n \times n$-matrix. If $C \cdot A = I_n$ for some matrix C, then $A \cdot \mathbf{x} = \mathbf{0}$ has only the trivial solution.

(38) Prove the following part of the Invertible Matrix Theorem: Let A be an $n \times n$-matrix. A is invertible iff A^T is invertible.

(39) Assume that $T\colon \mathbb{R}^n \to \mathbb{R}^n, x \mapsto A \cdot x$ is bijective. Show that A is invertible.
Hint: Use that T is onto $\mathbb{R}^n$ and the Invertible Matrix Theorem.

(40) Are the following matrices invertible? You do not need to compute the inverse. Just argue why or why not.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 0 & 0 \\ 4 & 2 & 3 \end{bmatrix}$$

(41) Can a square matrix with 2 identical rows be invertible? Why or why not?

(42) Are the following mappings invertible? If so, give their inverses.

(a) $f\colon \mathbb{R} \to \mathbb{R}^2, x \mapsto \begin{bmatrix} 2x \\ 3x \end{bmatrix}$

(b) $g\colon \mathbb{R}^2 \to \mathbb{R}^2, \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 2x - 3y \\ -x + 2y \end{bmatrix}$

(43) Let T be the rotation of $\mathbb{R}^2$ around the origin by the angle φ counterclockwise. Is the standard matrix of T invertible? If so, write down a formula for T^{-1}. What is its geometric interpretation?

(44) Are the following true or false? Explain why.

(a) Assume A implies B and B implies C. Then A implies C.

(b) A implies B and B implies A means that A is true whenever B is true, and A is false whenever B is false.

(c) n is an even integer $\Leftrightarrow$ $n + 1$ is an odd integer

(d) For $x, y \in \mathbb{R}$, $xy = 0$ iff $x = 0$ and $y = 0$.

(45) Give the negations of the following statements:

(a) $A \Rightarrow B$

(b) If you do well on the homework, you'll pass the class.

(c) $A \Leftrightarrow B$

(d) $x \in \mathbb{R}$ has an inverse if and only if $x \neq 0$.