Math 3130 - Assignment 4

Due February 12, 2016
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(28) Let $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T\colon \mathbb{R}^3 \to \mathbb{R}^2$, $\mathbf{x} \mapsto \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \mathbf{x}$.

  (a) Find the solution set of $T(\mathbf{x}) = \mathbf{b}$ in parametric vector form.

  (b) Give 2 vectors in $\mathbb{R}^3$ which are mapped to $\mathbf{b}$ by T, and give 2 vectors in $\mathbb{R}^3$ which are not mapped to $\mathbf{b}$ by T.

(29) Let T be as in (28) and let A be the standard matrix of T.

  (a) Do the colums of A span $\mathbb{R}^2$?

  (b) Are the columns of A linearly independent?

  (c) Is T injective? Is T surjective? Is T bijective?

(30) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear map such that

$$T(\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, T(\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

  (a) Express $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ as linear combination of $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.

  (b) Use the linearity of T to find $T(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix})$ and $T(\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix})$.

(31) Let $r, s \in \mathbb{R}$ and A, B be $m \times n$ matrices. Complete the proof of the following statement:

$$(r + s)(A + B) = rA + sA + rB + sB$$

Proof. Let $i \in \{1, \dots, m\}$ and $j \in \{1, \dots, n\}$. Then

$$\begin{aligned} ((r+s)(A+B))_{ij} &= \quad \dots && \text{by rule (b)} \\ &= \quad \dots && \text{by rule (a)} \\ &= \quad \dots && \text{by distributivity of real numbers} \\ &= \quad \dots && \text{by rule (b)} \\ &= (rA + sA + rB + sB)_{ij} && \text{by rule (a)} \end{aligned}$$

Thus $(r + s)(A + B) = rA + sA + rB + sB$. □

For the proof only use the following rules:

  (a) $(C + D)_{ij} = C_{ij} + D_{ij}$ for all matrices C and D of the same size,

  (b) $(tC)_{ij} = tC_{ij}$ for every scalar t and every matrix C.

(32) Let

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \ \mathbf{a}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \ \mathbf{a}_3 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \ \mathbf{a}_4 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

  (a) Find $x_1, x_2, x_3, x_4 \in \mathbb{R}$ such that $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 + x_4\mathbf{a}_4 = \mathbf{b}$. Verify your solution.

(b) Let $: \mathbb{R}^3 \to \mathbb{R}$ be a linear mapping such that $T(\mathbf{a}_1) = 10$, $T(\mathbf{a}_2) = 6$, $T(\mathbf{a}_3) = 8$, $T(\mathbf{a}_4) = 26$. Compute $T(\mathbf{b})$.

(33) Let 0 be the 0-matrix with size 2×2. Find 2×2 matrices $A \neq 0$ and $B \neq 0$ such that $AB = 0$.

(34) (a) If possible, invert the following matrices:

$$A = \begin{bmatrix} 2 & -3 \\ 4 & -9 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 2 \\ 4 & -3 \end{bmatrix}$$

(b) For which $a \in \mathbb{R}$ can the following matrix be inverted? Compute the inverse of C.

$$C = \begin{bmatrix} a-2 & 1 \\ 4 & a \end{bmatrix}$$

(35) If possible, invert the following matrix:

$$D = \begin{bmatrix} -3 & 2 & 4 \\ 0 & 1 & -2 \\ 1 & -3 & 4 \end{bmatrix}$$

(36) If possible, invert the following matrix:

$$E = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ -1 & 2 & -1 \end{bmatrix}$$

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.