Math 3130 - Assignment 2

Due January 29, 2016
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(10) [1, Section 1.4, Ex 17] How many rows of A contain a pivot position? Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for each $\mathbf{b} \in \mathbb{R}^4$?

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & -1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$$

(11) [1, Section 1.4, Ex 31] Let A be a 3×2 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all $\mathbf{b} \in \mathbb{R}^n$.

(12) Let $\mathbf{u} \in \mathbb{R}^n$ be a vector and let $c, d \in \mathbb{R}$ be scalars. Show that

$$(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}.$$

(13) [1, cf. Section 1.5, Ex 17] Let

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 8 \\ -16 \\ 12 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the equations $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$. Express both solution sets in parametric vector form. Give a geometric description of the solution sets.

(14) [1, cf. Section 1.5, Ex 11] Let

$$A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the equations $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$. Express both solution sets in parametric vector form.

(15) [1, Section 1.5, Ex 31] Let A be a 3×2 matrix with 2 pivot positions.
  (a) Does the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?
  (b) Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for every possible $\mathbf{b} \in \mathbb{R}^3$?

Explain your answers!

(16) [1, Section 1.7, Ex 9] Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}.$$

  (a) For which values of h is $\mathbf{w}$ in Span$\{\mathbf{u}, \mathbf{v}\}$?
  (b) For which values of h is $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ linearly dependent?

(17) [1, cf. Section 1.7, Ex 21] Mark each statement True or False, and justify each answer.
  (a) The columns of a matrix A are linearly independent if $\mathbf{x} = \mathbf{0}$ is a solution of $A\mathbf{x} = \mathbf{0}$.
  (b) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent, then each vector is a linear combination of the other two vectors.
  (c) The columns of any 4×5 matrix are linearly dependent.

(d) If $\mathbf{u}$ and $\mathbf{v}$ are linearly independent, and if $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent, then $\mathbf{w}$ is in the span of $\mathbf{u}, \mathbf{v}$.

(18) Show the following Theorem in 2 steps: Suppose $A\mathbf{x} = \mathbf{b}$ has a solution $\mathbf{p}$. Then the set of all solutions of $A\mathbf{x} = \mathbf{b}$ is

$$\mathbf{p} + \operatorname{NullSpace} A = \{\mathbf{p} + \mathbf{v} \mid \mathbf{v} \in \operatorname{NullSpace} A\}.$$

Suppose $A\mathbf{x} = \mathbf{b}$ has a solution $\mathbf{p}$.

(a) Show that if $\mathbf{v}$ is in $\operatorname{NullSpace} A$, then $\mathbf{p} + \mathbf{v}$ is also a solution for $A\mathbf{x} = \mathbf{b}$.

(b) Show that if $\mathbf{q}$ is a solution for $A\mathbf{x} = \mathbf{b}$, then $\mathbf{q} - \mathbf{p}$ is in $\operatorname{NullSpace} A$.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.