Math 3130 - Assignment 1

Due January 22, 2016
Markus Steindl

Solve all systems of linear equations by row reduction (Gaussian elimination).

(1) Do the following 4 planes intersect in a point? Which?

$$\begin{aligned} x + 5y + 3z &= 16 \\ 2x + 10y + 8z &= 34 \\ 4x + 20y + 15z &= 67 \\ x + 6y + 5z &= 21 \end{aligned}$$

(2) Add an equation of a line to the equation

$$2x + 3y = 4$$

such that the resulting system has (a) no solution, (b) exactly one solution, (c) infinitely many solutions.

(3) For which values $a \in \mathbb{R}$ does the following system of linear equations have more than one solution?

$$\begin{aligned} x + 2y &= 0 \\ 2x + ay &= 0 \end{aligned}$$

(4) First transform to echelon form, then to reduced echelon form. Solve the corresponding system of linear equations.

$$\begin{bmatrix} 0 & 0 & 2 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{bmatrix}$$

(5) First transform to echelon form, then to reduced echelon form. Solve the corresponding system of linear equations.

$$\begin{bmatrix} 2 & 2 & 1 & 8 & 2 \\ 2 & 0 & 0 & 8 & 2 \\ 2 & 6 & 3 & 8 & 1 \end{bmatrix}$$

(6) First transform to echelon form, then to reduced echelon form. Solve the corresponding system of linear equations.

$$\begin{bmatrix} 0 & 0 & 3 & 1 & 0 \\ 1 & 2 & 2 & -1 & 1 \\ 0 & 4 & 5 & -2 & 0 \\ 2 & 0 & 2 & 1 & 2 \\ -1 & 2 & 6 & 0 & -1 \end{bmatrix}$$

(7) [1, Section 1.3, Ex 12] Is $\mathbf{b}$ a linear combination of the vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$?

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \ \mathbf{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, \ \mathbf{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

(8) [1, Section 1.3, Ex 16] For which values of h is $\mathbf{y}$ in the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$?

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}, \ \mathbf{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$$

(9) Are the following true or false? Explain your answers.
- (a) Any system of linear equations with strictly less equations than variables has infinitely many solutions.
- (b) Different sequences of elementary row reductions can transform one matrix to distinct matrices in row echelon form.
- (c) The vector $3\mathbf{v_1}$ is a linear combination of the vectors $\mathbf{v_1}, \mathbf{v_2}$.
- (d) For $\mathbf{v_1}, \mathbf{v_2} \in \mathbb{R}^3$, $\text{Span}\{\mathbf{v_1}, \mathbf{v_2}\}$ is always a plane through the origin.

References

[1] David C. Lay. Linear Algebra and Its Applications. Addison-Wesley, 4th edition, 2012.