University of Colorado Boulder
Math 5001, Midterm, Part 2, Take-Home

Fall 2019

NAME: _____

HONOR CODE: On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance.

SIGNATURE: _____

Question	Points	Score
1	20	
2	20	
Total:	40	

- Only the course textbook by Rudin, lecture notes and your HW solutions are allowed.

- Read instructions carefully. Show all your reasoning and work for full credit.

- You can type or handwrite your answers. Copy the statement of each problem.

1. (20 points) If $\sum_n a_n$ and $\sum_n b_n$ are two series of nonnegative real numbers, prove that $\sum_n a_n$ and $\sum_n b_n$ converge if and only if $\sum_n \sqrt{a_n^2 + b_n^2}$ converges.

2. (20 points)   (a) (15pts) Prove or disprove: If $\{f_n\}$ is a sequence of continuous functions converging uniformly on $[0, 1]$ to f, then

$$\lim_{n \to \infty} \int_0^{1 - \frac{1}{n}} f_n(x)dx = \int_0^1 f(x)dx.$$

(b) (5pts) Prove or disprove: If each $f_n \geq 0$ on $[0, 1]$ and $\{f_n\}$ is a sequence of continuous functions such that $\sum_n f_n$ converges uniformly on $[0, 1]$ to f, then

$$\sum_{n=1}^{\infty} \int_0^{1 - \frac{1}{n}} f_n(x)dx = \int_0^1 f(x)dx.$$