Math 4001-5001: HW7

Due Friday, 10/25/2019

Problem 7.1

a) Finish the proof of the lemma from class: compact metric space K is separable. Show the set S we constructed is dense.

b) Deduce that for any $r > 0$, $K \subset \cup_{i=1}^{n} B_r(x_i)$, for some $x_i \in S$.

Problem 7.2

a) Let X be a compact metric space. Show it is bounded. (Hint: construct a clever open cover).

b) Let K be a compact metric space. If A is a compact set in $C(K)$, show A is equicontinuous. (Do a direct proof using definition of compactness.)

Note, in class we showed: Let K be a compact metric space. If A is a closed, bounded and equicontinuous set in $C(K)$, then A is compact.
Hence, this HW problem together with Theorem 2.34 completes the statement of the Arzela-Ascoli Theorem.

Problem 7.3

Let M be a positive finite number. Consider the following set

$$U = \overline{\{F(x) = \int_0^x f(t)dt : x \in [0,1], f \in \mathcal{R}([0,1]), ||f||_\infty \leq M\}}^{C([0,1])}.$$

a. Is $U \subset C([0,1])$?

b. Is U bounded in C([0,1])?

c. Is U equicontinuous?

d. Is U compact?

Prove your answers.