Math 4001-5001: HW5

Due Friday, 10/04/2019

Problems from the book:
page 165 # 1, 2.

Problem 5.1
For each of the following sequences, determine if the sequence converges pointwise or uniformly on a given set. Prove your conclusions; if a sequence does not converge uniformly, but only pointwise, prove it converges pointwise, and why it does not converge uniformly. Note: $\chi_E(x) = 1$ if $x \in E$ and $\chi_E(x) = 0$ if $x \notin E$ (χ_E is called a characteristic function of E).

a) $f_n(x) = \frac{\cos(2x)}{n} + \sin x$ on $E = [0, 1]$

b) $f_n(x) = \chi_{[n,n+1]}(x)$ on $E = [0, \infty)$

c) $f_n(x) = \frac{1}{n}\chi_{[0,n]}(x)$ on $E = [0, \infty)$

Problem 5.2

a) Recall the definition from class of $C([0, 1])$. Show the following is a norm on $C([0, 1])$

$$\|f\|_1 = \int_0^1 |f(x)|dx.$$

b) By considering the following sequence of functions show $\left(C([0, 1]), \|\cdot\|_1\right)$ is not a Banach space:

$$f_n(x) = 0 \text{ on } [0, \frac{1}{2}], \quad f_n(x) = n(x - \frac{1}{2}) \text{ on } (\frac{1}{2}, \frac{1}{2} + \frac{1}{n}], \quad f_n(x) = 1 \text{ on } (\frac{1}{2} + \frac{1}{n}, 1].$$

Extra Problems page 165 # 3, 12