Math 4001-5001: HW4

Due Friday, 9/27/2019

Problem 4.1
Prove Theorem 3.42.

Problem 4.2
In this problem we investigate a rearrangement of conditionally convergent series. First recall the rearrangement $\{k_n\}$ of natural numbers

$$1, 2, 4, 3, 6, 8, 5, 10, 12, \ldots.$$

In words: every odd number is followed by two adjacent even numbers.

a. Use $\{k_n\}$ to rearrange the alternating harmonic series

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots. \tag{1}$$

(Just write down the first few terms; you do not need a general formula for each a'_n.)

b. Derive formulas for partial sums of the rearrangement: show using induction that for $n \geq 3$ written as $n = 3k + \ell$, where $k \geq 1$ and $\ell = 0, 1$, or 2, the partial sum of the rearrangement is given by

$$s_n = 1 - \frac{1}{2} - \frac{1}{4} + \cdots + \frac{1}{2k-1} - \frac{1}{4k-2} - \frac{1}{4k} + r_n,$$

where the remainder r_n consists of the next $\ell (= 0, 1, 2)$ terms in the partial sum.

c. Show r_n goes to zero as $n \to \infty$.

d. Group the terms of the partial sum (the associative law applies!) so you can show that for $n = 3k + \ell$ as above

$$s_n = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} + \cdots + \frac{1}{4k-2} - \frac{1}{4k} + r_n. \tag{2}$$

e. Within s_n in the equation (2) find a partial sum of (1) (The word "within" is purposely vague, so you can be more creative.). Take n to infinity. What does the rearrangement of the series in (1) converge to?

Problem 4.3
Prove Theorem 3.55: apply a direct ϵ and N argument to show that s'_n converges to $s = \sum a_n$.

Problem 4.4
Prove Theorem 7.9.