Math 4001-5001: HW2

Due Friday, 9/13/2019

Problem 2.1

Discuss the convergence or divergence of the following series (you can assume all the series start at $n = 1$):

a. $\sum a_n$, $a_n = \frac{1}{1+n^2}$

b. $\sum a_n$, $a_n = \frac{1}{1+n}$

c. $\sum a_n$, $a_n = \frac{1}{3^n-1}$

d. $\sum a_n$, $a_n = \frac{1}{2n^3-29}$

Problem 2.2

Suppose the series $\sum c_n$ and $\sum d_n$ converge to the same value s. Show that if

$$\sum_{n=1}^{N} c_n \leq \sum_{n=1}^{N} a_n \leq \sum_{n=1}^{N} d_n,$$

for all $N \geq 1$, then the series $\sum a_n$ also converges, and it converges to s.

Problem 2.3

If the a_n form a nonincreasing sequence of nonnegative numbers, i.e., $a_n \geq a_{n+1} \geq 0$, and d is a fixed *natural number* such that $d > 1$, show that $\sum_{n=1}^{\infty} a_n$ converges if and only if the series $\sum_{k=0}^{\infty} d^k a_{d^k}$ converges.

Problem 2.4

Let l_1 be the set of real valued sequences $\{x_n\}$ such that $\sum_{n=1}^{\infty} |x_n|$ is convergent, and l_2 be the set of real valued sequences $\{x_n\}$ such that $\sum_{n=1}^{\infty}(x_n)^2$ is convergent, i.e.,

$$l_1 = \left\{ \{x_n\} \subset \mathbb{R} : \sum_{n=1}^{\infty} |x_n| < \infty \right\}$$

$$l_2 = \left\{ \{x_n\} \subset \mathbb{R} : \sum_{n=1}^{\infty} (x_n)^2 < \infty \right\}$$

Show $l_1 \subset l_2$.