Math 4001-5001: HW12

Due Friday, 12/06/2019

Problem 12.1 If ϕ is an additive set function show

  a. $\phi(\emptyset) = 0$.

  b. $\phi(A_1 \cup A_2) + \phi(A_1 \cap A_2) = \phi(A_1) + \phi(A_2)$.

  c. If ϕ is nonnegative, i.e., $\phi(A) \geq 0$ for every A, and $A_1 \subset A_2$, then

$$\phi(A_1) \leq \phi(A_2).$$

  d. If $B \subset A$ and $|\phi(B)| < \infty$, then $\phi(A - B) = \phi(A) - \phi(B)$.

Problem 12.2 Let G be an open subset of $\mathbb{R}^n$. Show $m_*(G) = m^*(G) = m(G)$ where $m(G)$ is the measure defined in step 3.

Problem 12.3

  a. Show a closed line segment has n dimensional Lebesgue measure equal to 0 for $n \geq 2$.

  b. Show $\mathbb{Q}$ has Lebesgue measure 0.