Math 4001-5001: HW11

Due Friday, 11/22/2019

Problem 1 The purpose of this problem is to show the continuity of f' at a point a is needed in the Inverse Function Theorem even in $1D$. Let

$$f(t) = t + 2t^2 \sin(\frac{1}{t}), \quad t \neq 0,$$

and $f(0) = 0$. Show

a. $f'(t)$ exists for all $t \in \mathbb{R}$.

b. f' is invertible at $t = 0$.

c. f' is not continuous at 0, and it is continuous if $t \neq 0$. (You can quote that the limit of $\cos(\frac{1}{t})$ does not exist as $t \to 0$.)

d. f is not $1 - 1$ in any neighborhood of 0.

Problem 2 Consider the following system of equations:

$$x_1^2 + x_2^2 + y^2 = 1,$$
$$x_1 + x_2 + y = 1.$$

Can we apply the Implicit Function Theorem to solve for $x = (x_1, x_2)$ in terms of y in the neighborhood of $(1, 0, 0)$, i.e., does there exist a mapping $g : W \to U$, where $0 \in W$ and $(1, 0) \in U$, such that $x = g(y)$ and the above equations are satisfied? Address each hypothesis in the statement of the theorem.

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