Math 4001-5001: HW1

Due Friday, 9/06/2019

Problems from the book:
p. 23: 13 (instead of x, y being complex suppose $x, y \in \mathbb{R}^k, k \geq 1$.)
p. 78: 1, 20

Problem 1.1

a. Let $x \in \mathbb{R}^k$ and define $||x||_1 = \sum_{i=1}^{k} |x_i|$. Show $|| \cdot ||_1$ is a norm on $\mathbb{R}^k$.

b. Show $\sum_{i=1}^{k} |x_i| \leq \sqrt{k}|x|$ for any $x = (x_1, \cdots, x_k)$.

c. If X is a vector space and $|| \cdot ||$ and $|| \cdot ||_2$ are two norms on X, they are said to be *equivalent* if there exist constants $c, d > 0$ such that

$$c||x|| \leq ||x||_2 \leq d||x|| \quad \text{for every } x \in X.$$

Show $|| \cdot ||_1$ is equivalent to the standard Euclidean norm on $\mathbb{R}^k$.

Problem 1.2
Suppose X is a normed linear space with a norm $|| \cdot ||$. Show X is a metric space.

Problem 1.3
Let $k \geq 2$. Use that $\mathbb{R}$ is complete to show $\mathbb{R}^k$ is complete.