Math 4001-5001: Final Topics

The exam will consist of five questions:

1. Infinite Series: Series starting on page 58 up to Theorem 3.28 (statements and proofs), review HW problems, Midterm;
2. Arzela-Ascoli Theorem: know the statement; HW problems;
3. Differentiation: 9.11 up to the statement of Theorem 9.21; HW problems; proving something satisfies the definition of the derivative;
4. Inverse Function Theorem/Implicit Function Theorem: HW problems;
5. Lebesgue Measure & Integration: HW problems, and
   (a) Knowing how to show $m(\mathbb{R}^n) = \infty$.
   (b) The following properties of Lebesgue measure:
      i. If $A \in \mathcal{L}$, then $A^c \in \mathcal{L}$.
      ii. Countable unions and countable intersections of measurable sets are measurable.
      iii. If $A, B \in \mathcal{L}$, then $A - B \in \mathcal{L}$.
      iv. If $A_k \in \mathcal{L}$, then $m(\cup_{k=1}^{\infty} A_k) \leq \Sigma_{k=1}^{\infty} m(A_k)$ (i.e., m is countably subadditive) and if A_k are pairwise disjoint, then
      $$m(\cup_{k=1}^{\infty} A_k) = \Sigma_{k=1}^{\infty} m(A_k), \text{(i.e., } m \text{ is countably additive).}$$
      v. All open sets and all closed sets are measurable.
      vi. If $m^*(A) = 0$, then A is measurable and $m(A) = 0$. (know how to show this)
      vii. If A is measurable, then $m^*(A) = m_*(A) = m(A)$.
   (c) Know how to integrate simple functions (see examples from class).
   (d) Definition of Lebesgue integral for a general measurable function.

Good Luck!