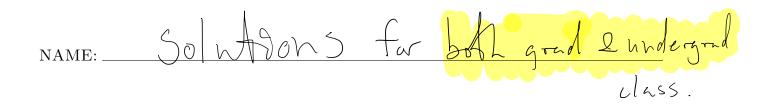
University of Colorado BoulderMath 5001, Midterm, Part 2, Take-Home $\mathcal{L} \ M \mathcal{A} \ 100$ Fall 2019



Question	Points	Score
1	20	
2	20	
Total:	40	

- Only the course textbook by Rudin, lecture notes and your HW solutions are allowed.
- Read instructions carefully. Show all your reasoning and work for full credit unless indicated otherwise.
- On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance.
- You can type or handwrite your answers. Copy the statement of each problem.

- 1. (20 points) If $\sum_{n} a_{n}$ and $\sum_{n} b_{n}$ are two series of nonnegative real numbers, prove that $\sum_{n} a_{n}$ and $\sum_{n} b_{n}$ converge if and only if $\sum_{n} \widehat{a_{n}^{2} + b_{n}^{2}}$ converges.
- 2. (20 points) (a) (15pts) Prove or disprove: If $\{f_n\}$ is a sequence of continuous functions converging uniformly on [0, 1] to f, then

$$\lim_{n \to \infty} \int_0^{1 - \frac{1}{n}} f_n(x) dx = \int_0^1 f(x) dx.$$

(b) (5pts) Prove or disprove: If each $f_n \ge 0$ on [0, 1] and $\{f_n\}$ is a sequence of continuous functions such that $\sum_n f_n$ converge uniformly on [0, 1] to f, then

$$\sum_{n=1}^{\infty} \int_{0}^{1-\frac{1}{n}} f_{n}(x)dx = \int_{0}^{1} f(x)dx.$$

4. By HW Broblem 1.1 c), we have \exists constants
 $c_{1} d > 0$ such that if $x = (a_{1}b)$ then
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 $c_{1} d > 0$ such that $if x = (a_{1}b)$ then
 $c_{1} d > 0$ such that $directly now$: for this problem we need
 $0 \le a_{1}b \le \sqrt{a^{2}+b^{2}} \le a + b$
 $0 \le a_{1}b \le \sqrt{a^{2}+b^{2}} \le a + b$
By given $0 \le a_{1}b$. Also $a = |a| = \sqrt{a^{2}} \le \sqrt{a^{2}+b^{2}}$ since $b^{2} \ge 0$.
By given $0 \le a_{1}b$. Also $a \ge |a| = \sqrt{a^{2}} \le \sqrt{a} = \sqrt{x}$ are increasing
functions for $x \ge 0$.
Now suppose $\sum a_{1}b \ge a^{2}+b^{2} \le (a + b)^{2}$ $x^{2} \ge \sqrt{x}$ are increasing
the could use This 3.24 , but we will just use the
comparison text.
First $\sum a_{n}+b_{n}$ converges (do a direct proof or guidle This
 3.47)
Then $|\sqrt{a_{n}+b_{n}}| \ge \sqrt{a_{n}+b_{n}} \le a_{n}+b_{n} \forall n by (39)$.
Then $|\sqrt{a_{n}+b_{n}}| \ge \sqrt{a_{n}+b_{n}} \le a_{n}+b_{n} \forall n by (39)$.

Now suppose
$$\sum \sqrt{a_n^2 + b_n^2}$$
 converges.
By (2) $|a_n| = a_n \leq \sqrt{a_n^2 + b_n^2}$.
So $\sum a_n$ converges by the comparison test.
Submilarly, the same holds for $\sum b_n$ since $|b_n| \leq \sqrt{a_n^2 + b_n^2}$.

$$\begin{array}{c} 2 \\ \hline \\ & \\ \hline \\ \\ \hline \\ \\ & \\ \hline \\ \\ \hline \\ \\ \\ \hline \\ \\ \hline \\$$

Hence

$$|\int_{0}^{1-t_{n}} f_{n} - \int_{0}^{1} f | \leq |\int_{0}^{1-t_{n}} f_{n} - \int_{0}^{1} f_{n}| + |\int_{0}^{1} f_{n} - \int_{0}^{1} f |$$

$$\leq |\int_{1-t_{n}}^{1} f_{n}|$$

$$\leq H t_{n} \rightarrow 0 \text{ as } h \rightarrow b$$

(b) (5pts) Prove or disprove: If each $f_n \ge 0$ on [0, 1] and $\{f_n\}$ is a sequence of continuous functions such that $\sum_n f_n$ converge uniformly on [0, 1] to f, then

$$\sum_{n=1}^{\infty} \int_{0}^{1-\frac{1}{n}} f_{n}(x) dx = \int_{0}^{1} f(x) dx.$$
Proof: The statement is false.
(on states $f_{n}(x) = \frac{1}{2^{n}} \chi_{[0,1]}$ then $\sum_{n=1}^{\infty} f_{n}(x)$ converges
uniformly to $f(x) = \chi_{[0,1]}$ by the geometric series.
We then have $\sum_{n=1}^{\infty} \int_{0}^{1} \frac{1}{2^{n}} dx = \int_{0}^{1} \frac{1}{2^{n}} dx = \int_{0}^{1} 1 dx = 1$ by $HU/(corollary p.152)$.

Now consider

$$\sum_{n=1}^{\infty} \int_{0}^{1-\frac{1}{n}} f_{n}(x) dx = \int_{0}^{1-\frac{1}{2}} \frac{1}{2} dx dx + \sum_{n=2}^{\infty} \int_{0}^{1-\frac{1}{n}} \frac{1}{2^{n}} dx$$

$$= \sum_{n=2}^{\infty} \int_{0}^{1-\frac{1}{n}} \frac{1}{2^{n}} dx \neq 1$$

$$= \int_{0}^{1-\frac{1}{2}} \frac{1}{2} dx = \frac{1}{2} \quad \& \int_{0}^{1-\frac{1}{2}} \frac{1}{2} dx = 0$$