Main Concepts:

Defn of an ODE

Defn of an order of an ODE

Stability & direction fields

Existence & Uniqueness Thm: not iff

the thm does not apply

if the hypothesis fails

What does it mean to be a soln:

$y \in C^1(I)$

& interval of existence

Methods:

0. $y' = f(x)$

1. separable

$y' = g(x)h(y)$

$\frac{dy}{h(y)} = g(x)\,dX$

subtleties / difficulties

What is $\int f(x)\,dx$?

dividing be zero

y: Case 1: $h(y) \neq 0$ Case 2: $h(y) = 0$

x: $g(\tilde{x})$ is undefined, then $\tilde{x} \notin I$.

IVP: $y(x_0) = y_0$

Choose largest I so that $x_0 \in I$ & $g(x)$ is defined on I.

And in the end, y is also defined on I.

General: choose I so that $g(x)$ & y are defined.

integrating factor

linear

$y' + py = q$

$e^{\int p(x)\,dx}$

If $p(\tilde{x})$ or $q(\tilde{x})$ are undefined, $\tilde{x} \notin I$.

IVP: same idea as above p, q, y defined & $x_0 \in I$.

general: p, q defined & y defined.

substitution methods

homogeneous: $v = \frac{y}{x}$

Bernoulli: $v = y^{1-\alpha}$

How to tell which one?

- If you see $\cos x$, e^x in the ODE, probably not homogeneous
- Homogeneous: some form of a polynomial + check total exponent

$$y' = \frac{x^2 y + y^2 x}{y x^2 - y^3}$$

- Bernoulli:

$$y' + p(x) y = q(x) y^{\alpha}$$

$\cancel{\cos(y^2)}$

Exact

Check: $\partial_y M = \partial_x N$

$$M + N y' = 0$$

equivalent to

$$M\,dx + N\,dy = 0$$