## 2 Examples

### 2.1 Find the general solution of the following ODE.

a) $2y + 2xy' = 0$

Step 1: $M(x,y) = 2y$ $N(x,y) = 2x$

Exact? $\partial_y M = 2 = \partial_x N$

Step 2:

Method 1: $\Phi(x,y) = \int 2y \, dx + g(y)$

$= 2xy + g(y)$

$\partial_y \Phi = 2x + g'(y) = N = 2x$

$g'(y) = 0$

$g(y) = \tilde{C}$

step 3: $\Phi(x,y) = C$

$2xy + \tilde{C} = C$

or $2xy = C$

Method 2:

$$\Phi(x,y) = \int N(x,y)\,dy + h(x)$$
$$= \int 2x\,dy + h(x)$$
$$= 2xy + h(x)$$

$$\partial_x \Phi = 2y + h'(x) = M = 2y$$

$$h'(x) = 0$$
$$h(x) = \tilde{c}$$

$$\Phi(x,y) = c$$
$$2xy = C$$

Method 3:

$$2xy + g(y) = \Phi$$
$$2xy + h(x) = \Phi$$

b) $[1+\cos(x+y)]y' = -\cos(x+y)$

Step 1:

$M = \cos(x+y)$

$N = 1 + \cos(x+y)$

$\partial_y M = -\sin(x+y) = \partial_x N$ ✓

Step 2:

$\Phi(x,y) = \int \cos(x+y)\,dx + g(y)$

$= \sin(x+y) + g(y)$

$\partial_y \Phi = \cos(x+y) + g'(y) = N$

$= 1 + \cos(x+y)$

$g'(y) = 1$

$g(y) = y + \tilde{C}$

Step 3:

$\Phi(x,y) = C$

$$\boxed{\sin(x+y) + y = C}$$

c) $3x^2y^2 + ye^{xy} + (2x^3y + xe^{xy} + \cos y)y' = 0$

$M(x,y) = 3x^2y^2 + ye^{xy}$

$N(x,y) = 2x^3y + xe^{xy} + \cos y$

$\partial_y M = 6x^2y + e^{xy} + yxe^{xy}$

$\partial_x N = 6x^2y + e^{xy} + xye^{xy}$

Exact!

$\Phi(x,y) = \int M(x,y)\,dx + g(y)$

$= \int 3x^2y^2 + ye^{xy}\,dx + g(y)$

$= x^3y^2 + e^{xy} + g(y)$

$\partial_y \Phi = 2x^3y + xe^{xy} + g'(y) = N(x,y)$

$= 2x^3y + xe^{xy} + \cos y$

$\Rightarrow g'(y) = \cos y$

$g(y) = \sin y + C_1$

$\Phi(x,y) = C$

$\boxed{x^3y^2 + e^{xy} + \sin y = C}$

### 2.2 Solve the IVP

$2y + 2xy' = 0, \quad y(-1) = 1.$

From a)

$2xy = C$

Need $y(-1) = 1$

$2(-1)(1) = C$

$C = -2$

$2xy = -2$

$xy = -1$

$y = -\frac{1}{x}$ $\quad I = (-\infty, 0)$

$x \neq 0$