# 1 What does it mean to be a solution of an ODE?

In class, we introduced the following function spaces. Let $J$ be an interval, then

$$C(J) = \{f : J \to \mathbb{R} : f \text{ is continuous on } J\}, \tag{1}$$

$$C^k(J) = \{f : J \to \mathbb{R} : f, f', \dots, f^{(k)} \text{ are continuous on } J\}, \tag{2}$$

$$C^\infty(J) = \{f : J \to \mathbb{R} : f^{(k)} \text{ is continuous on } J \text{ for any } k \geq 0\}. \tag{3}$$

Then we gave a definition of a solution.

**Definition 1.** *Given a k'th order ODE, a function $y$ is a (classical) solution to the ODE on an interval $J$ if $y \in C^k(J)$ and $y$ satisfies the ODE identically on $J$.*

**Remark 1.1.** *So when we are asked to solve an ODE, we should give a function $y$ and interval $J$. We will follow this when we are solving an (IVP).*

**Remark 1.2.** *The interval $J$ is called an interval of existence for the solution $y$. The goal is to give the maximal (largest possible) interval of existence.*

**Remark 1.3.** *The interval $J$ is going to be just one interval; it cannot be a union of intervals. This means that the interval of existence is not going to be always the same as the domain of the function $y$, when $y$ is thought of only as a function in a Calculus sense. For us $y$ is a function that solves an ODE: so it is a function in a Calculus sense, but it also has an additional property of solving an ODE. That additional property forces restrictions on where $y$ should be defined. As a result, the interval of existence can be strictly smaller than what the domain of $y$ would be if we were just thinking about it as a function.*

**Remark 1.4.** *We need $y$ to be $C^k$ on $J$, but we also need the ODE to be well-defined on $J$. Therefore, the search for the interval $J$ starts with looking at the ODE and seeing if there are any points (say, $x$) for which the ODE might not be defined.*

### Intervals of existence for the HW problems

Section 1.4 Exercise 2a) $J = (-\infty, \infty)$
Section 1.4 Exercise 2b) $J = (0, \pi)$
Section 1.4 Exercise 2c) $J = (-\infty, 0)$. Here, we can give the answer for $y$ as $y = \frac{\ln|t|-1}{t}$ as the book or $y = \frac{\ln(-t)-1}{t}$