Math 3430

Quiz # 5

NAME: Solution

Problem 1. True/False: The following ODE is exact. Show work.

$$(3x^2y^2 + ye^{xy})dx + (2x^3y + xe^{xy} + \cos y)dy = 0$$

$M(x,y) = 3x^2y^2 + ye^{xy}$

$N(x,y) = 2x^3y + xe^{xy} + \cos y$

$\partial_y M = 6x^2y + e^{xy} + xye^{xy}$

$\partial_x N = 6x^2y + e^{xy} + xye^{xy}$

$\partial_x N = \partial_y M$

so Exact.

(Compare to Example c) on the exact eqns worksheet)

Problem 2. Which type of substitution would allow you to solve the following ODE? Make the substitution and rewrite the equation in terms of the new variable (v), but DO NOT SOLVE.

a)

$$y^{\cancel{2}\,4}e^xy' + y^{\cancel{3}\,5}\sin x = 1$$

$y' + y\sin x\, e^{-x} = e^{-x}y^{-4}$

$v = y^{1-(-4)} = y^5$

$v' = 5y^4y'$

$y = v^{\frac{1}{5}} \qquad y^4 = v^{\frac{4}{5}}$

$\dfrac{v'}{5v^{4/5}} + v^{\frac{1}{5}}\sin x\, e^{-x} = e^{-x}v^{-4/5}$

or $v' + 5v\sin x\, e^{-x} = 5e^{-x}$

b)

$$2xyy' = x^2 + 2y^2$$

$y' = \dfrac{x^2 + 2y^2}{2xy}$

$= \dfrac{x}{2y} + \dfrac{y}{x}$

$v = \dfrac{y}{x}$

$y = xv \qquad y' = v + xv'$

so

$v + xv' = \dfrac{1}{2v} + v$