Math 3470: HW 6

Due Friday, 10/11

Problem 6.1

Let J be an interval. Recall the following definition.

Definition 1. *An operator $L : C^\infty(J) \to C^\infty(J)$ is linear if*

$$L(af + bg) = aLg + bLg, \quad \forall a, b \in \mathbb{R}, \quad \textit{and} \quad \forall f, g \in C^\infty(J). \tag{1}$$

If (1) does not hold, L is called nonlinear.

Check directly using the Definition 1 above if L defined below are linear or nonlinear.

a) $L = a\frac{d^2}{dx^2} + b\frac{d}{dx} + c, \quad a, b, c \in \mathbb{R}$.

b) $L = \frac{d}{dx} + \ln|\cdot|$ (Note, this means $Ly = y' + \ln|y|$.)

c) $L = \frac{d^2}{dx^2} - \ln x$.

Problem 6.2

Prove the following theorem (see the set-up from lecture): Multiplication by a fixed function defines a linear operator.

Problem 6.3

Use Problem 6.1 to answer the following questions: Are the ODE below linear or nonlinear? Homogeneous or inhomogeneous.

a) $Ly = 0$, where L is the operator from Problem 6.1 a)

b) $Ly = 0$, where L is the operator from Problem 6.1 b)

c) $Ly = e^x$, where L is the operator from Problem 6.1 a)

d) $Ly = y$, where L is the operator from Problem 6.1 a)

e) $Ly = \ln|y| + \ln x$, where L is the operator from Problem 6.1 b)

Problem 6.4

Classify the differential equation in terms of order, linearity (circle nonlinear terms, if any), and homogeneity (underline inhomogeneous terms, if any). If the equation is linear, identify the linear operator L, and rewrite the ODE as $Ly = g$. You do not have to prove L is linear.

a) $y'' - y = 0$

b) $y' = \ln x$

c) $y' + (\sin x)y - 1 = 0$

d) $y'' + by' + cy = y^2$

e) $y'' + x^2y' - e^x = xy$