Prior projects: Prior to doing this project, students should have done these project:

- Introduction to the types of functions seen in Calc 3

Philosophy behind this project:
In this project, the concept of derivative is extended beyond what is presented in the textbook. The text focuses primarily on derivatives for functions from $\mathbb{R}$ to $\mathbb{R}^n$ or the partial derivatives for functions from $\mathbb{R}^m$ to $\mathbb{R}$. Our approach considers derivatives for differentiable functions from $\mathbb{R}^m$ to $\mathbb{R}^n$. We expand the scope of the concept of derivatives that is typically covered in Calculus 3, allowing us to explore the what "derivative" means for scalar-valued and vector-valued functions of multiple variables. This activity helps prepare students for a follow-up project which asks students to generalize the chain rule. A more general understanding of derivatives is required background in both differential equations and differential geometry.

Learning Goals:

1. Review of some trig, exponential, and logarithm derivatives
2. Review of partial derivatives
3. Recognize the form of derivatives for functions mapping from $\mathbb{R}^m$ to $\mathbb{R}^n$
4. Apply differentiation rules to compute derivatives for any differentiable function mapping from $\mathbb{R}^m$ to $\mathbb{R}^n$
5. Review what it takes for functions mapping from $\mathbb{R}^m$ to $\mathbb{R}^n$ to be differentiable
6. Incorporate concept of matrices to Calc 3 concepts

Implementation Notes:

1. This project should take students roughly 10 minutes to complete. The majority of the time will be spent reading the first page. The students will have seen the top half of the first page at the beginning of the semester.

2. This is an excellent opportunity to continue building students' vocabulary for discussing different types of functions. Emphasize students to discuss functions as scalar-valued function, parametric curve functions, etc.

3. It should be brought to students attention that for the derivatives of these functions to exist each partial derivative must exist and be continuous. This can be done either pointing this out to students while they work in groups, or stopping the groups to have a brief whole class discussion.

4. This may be one of the first time students think of functions as matrices. There should be an emphasis on discussing entry location for matrices, such as naming the first row, first column entry as a_{11}.

5. It may also be worth mentioning that we can think of these vector derivatives as either a single row matrix or single column matrix.

6. Ask students if they notice a relationship between the number of rows and columns and the dimension of the domain and codomain.

7. Page 1 includes a review of functions, and then has students write what their derivatives look like. Students are given a specific function and its derivative for vector-valued functions of multiple variables from $\mathbb{R}^3$ to $\mathbb{R}^2$. You may want to ask students what the derivative would look like for a general function from $\mathbb{R}^m$ to $\mathbb{R}^n$.

8. Page 2 is practice computing derivatives. Either have groups verify their derivatives are correct by checking with other groups, or have students present their solutions on the board for discussion.

Wrap-Up:

1. The wrap-up should be brief. Explain to students that they know who to find the derivative for any differentiable function they encounter from $\mathbb{R}^m$ to $\mathbb{R}^n$.

2. Students should be made aware of that some of this material is not included in the textbook. The text typically focuses on just the entries of a derivative matrix, and does not discuss the total derivative.

3. It is worth mentioning to students that a strong understanding of derivatives will help them generalize the derivative rules they learned in Calculus 1.

4. It may be worth mentioning to the students that they now know how to find first derivatives for these functions, that they may want to explore what the second derivatives look like on their own.