Background content: Prior to beginning the project, the instructor must give at least the definition of a vector field, and an explanation that the graph is created by substituting points into the formula and drawing the output vector starting at the given input point. It is valuable to give applications (force field, wind or water flow). With this minimal information, students are typically able to do the project and gain some intuition about the graphs of vector fields.

Philosophy behind this project: Direct instruction on vector fields poses problems since the graphs cannot be created easily in real time. The purpose of this project is to expose students to multiple examples and have them work with these examples without them or the instructors having to time-consumingly draw the graphs by hand.

Learning Goals:

1. The graph of a vector field can be created by plotting arrows one at a time. Substitute points in the plane to determine what arrow to draw at that point.
2. The length of the vectors in a vector field can be evaluated directly from the formula. These should match the lengths in the graph.
3. When graphing a vector field, we scale the length of the vectors so they don't overlap. Thus a graph of a vector field shows the relative length and not absolute length of the vectors.
4. Recognize that in a constant vector field, the arrows all point in the same direction.
5. Use a variety of strategies to determine general characteristics of a vector field, such as if the vectors point directly towards or directly away from the origin, or if lengths of the vectors remain constant, increase or decrease as we move away from the origin.
6. When presented with a formula for a function from $\mathbb{R}^3$ to $\mathbb{R}^3$, recognize that it can be depicted as a collection of arrows in 3-space.
7. Interpret a vector field as a force field, or a diagram of wind or water flow (this learning goal must be provided by the instructor during class.)

Implementation Notes: The project seems to work pretty smoothly and engage students' curiosity. You don't have to tell them too much before starting - the key is to draw out conclusions from the students to share with the class. Here are some questions to ask the students:

- Have you examined the coordinate system and determined where the axes and origin are?
- How do I recognize from a formula that the vectors all point directly away from the origin? What about directly towards the origin?
- How do I determine if the vectors all have the same length, if they become longer as I move away from the origin, or if they becomje shorter as I move away from the origin?
- How will the graph of $\mathbf{F}(x,y) = \langle 1,2\rangle$ differ from the graph of $\mathbf{F}(x,y) = \langle 3,6\rangle$? Why? (This is a tricky question. Though the fields are different, our graphs may end up looking identical, since we scale the length of the vectors so they don't overlap. We lose information by scaling, but it is done out of necessity to make the graph readable.
- How did you distinguish between the formulas for the clockwise-rotating and counter-clockwise-rotating fields?