Background content: Prior to doing this project, when given an equation in x, y and z, students should know how to substitute values for the each of the variables to determine the shape of cross-sections of the graph. They also need to know how to graph, based on their formulas, the conic sections that make up the cross-sections. These include hyperbolas, ellipses and parabolas. Lastly, they need to know the names of the various types of quadric surfaces.

Philosophy behind this project:
Our goal is for students to develop confidence and ease in graphing scalar functions of two variables by studying quadric surfaces. They will learn to relate equations with their three-dimensional graphs and with the graphs of their level curves. They will also practice naming quadric surfaces. The key skill in this process is finding and graphing cross-sectional curves.

Learning Goals:

1. Practice graphing equations in three-dimensions by finding the cross-sectional curves.
2. Practice naming quadric surfaces.
3. Practice visualizing the level curves of a three-dimensional graph.

Implementation Notes:

1. Notice that the graph of each of the surfaces shows the cross-sections perpendicular to both the x- and y-axes. Thus, by looking at both the graph of the surface and the graph of the level curves, we can see cross-sections across all three axes.
2. It's an option to hand out the cards without giving directions. Sometimes it switches their brains on if they have to figure out what they are supposed to do.
3. They may not have enough time to complete this exercise. If you have to manage the time, then one way is to ask them to first match the graphs of the surfaces with the graphs of the level curves, which is primarily a visualization exercise and not particularly time-consuming. When they complete that, ask them to match one equation at a time with the surfaces and level curves. This part of the activity practices calculating and graphing level curves, and cross-sections perpendicular to the x- and y-axes. Once they match an equation with its graph, then they can match that triple with the name of the quadric surface. By organizing the time in this way, even if the students do not finish, they will have practiced each part of the project, and can complete it independently at home. You can ask them to do specific equation cards if you want to make sure they do at least one elliptical paraboloid, one hyperbolic paraboloid and one hyperboloid.
4. The cross-sections of quadric surfaces are of course all conic sections. You may have to begin class by reviewing these.