Prior projects: Prior to doing this project, students should have done these project:

- Introduction to Mathematica and Graping Project
- Guidelines for 3D Graphing

Philosophy behind this project:
This project asks students to connect their understanding of Riemann sums from single variable calculus to double integrals. The intention of the project is to recognize the geometric difference between integrating with respect dA and $dxdy$ (or $dydx$).

Learning Goals:

1. Review concept of Riemann Sums from single variable calculus
2. Graphing solids from bounds of planar region and surface function
3. Examine Riemann Sums with double integrals
4. Approximating volume with Riemann approximation
5. Review iterated integrals
6. Review bounds of integration

Implementation Notes:

1. Page 1 asks students to draw a solid bounded by multiple planes.
   (a) As with other projects students work through this semester, the intention of this project is to extend students' single variable understanding to multi-variable case. It can be beneficial to give a brief review of the concept of Riemann sums as an introduction before the students begin the project.
   (b) Problem 2 can be tricky for some students since there is a small pyramid section below the xy-plane. It is important that all groups have the correct drawing before continuing to the second page to ensure there is a common starting point to think about approximating its volume.
2. Page 2 has students approximate the volume of the solid with four rectangular prisms.
   (a) The project does not give the specific corner or center of the rectangular prism intentionally.
   (b) The instructor should give each group different points to find the heights of the prisms, and then have the groups compare approximations.
   (c) There should be at least one group for the lower left corner, upper right corner, and center of rectangles.
   (d) Then there can be a discussion about which location would give the best approximation in this situation, and when other locations would give better approximations.
3. Page 3 transitions from Riemann sums for dA to iterated integrals $dx\,dy$. Students now examine the volume of a section of a paraboloid by thinking about slabs.
   (a) The slabs are first parallel with the yz-plane. Students may forget that they are not finding the area for the entire paraboloid, and ignore the region that they are finding the volume above.
   (b) Note that the language used to discuss a small section of volume is a slab in the project. These sections of volume may also be described as slices.
   (c) It should be pointed out how the order of integration impacts the way we create sections of volume to fill the entire solid.
4. Page 4 has two purposes: (1) Have students recognize that if they try to use just rectangular region to integrate over, they will not be able to calculate the volume of a paraboloid, and (2) Gives motivation for the need of integrating with a different coordinate system, in particular polar coordinates.
   (a) Do not have the students get too bogged down with evaluating their integral from Problem 12. The integration is intensive, and is not the point of the problem.
   (b) As soon as they realize that the integration will be rough, have them move on to the last problem and start thinking about better ways to calculate the volume of the paraboloid.
   (c) If there are students who complete the project early, there is a natural extension to ask students to create a geometric interpretation of integrating using polar coordinates to calculate volume.

Wrap-Up:

1. The wrap-up for this project will probably take 3-5 minutes. Start the wrap-up by getting students to explain how they've been connecting their understanding of single variable calculus to the new ideas they're encountering throughout semester. After the discussion gets going, if students have not explicitly mentioned Riemann sums, direct the conversation toward their work in this project.

2. Once the students begin making connections to previous ideas, push them to think about what is different. Now try to move students to describe the difference integrals with dA and iterated integrals. Included in this discussion should be the impact of order of integration.

3. If students get to the last problem, discuss the motivation for integrating using different coordinate systems.