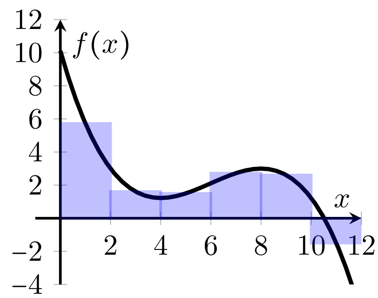
**Review of Calculus 1:**

Recall from single variable calculus that we are able to approximate the area under continuous curves on closed intervals by partitioning the intervals into $n$ subintervals of equal width $\Delta x$ to form Riemann sums.




To find the exact area, we refine our partition by adding more and more subintervals letting the width of each subinterval approach zero, resulting in the definite integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$$

**Extending Concept to Calculus 3:**

We now extend this idea to integrals of functions of two variables, we approximate the volume under a continuous surface on a closed planar region.

1. Draw the planar region defined by $R = [0,8]\times[0,4] = \left\{(x,y)\in\mathbb{R}^2 | 0 \le x \le 8, 0 \le y \le 4\right\}$ on $z = 0$ in 3-space.

Now we approximate the signed volume of the region under a continuous function of two variables and above the closed, bounded region $R$.

2. Draw the solid represented by $\displaystyle\iint_R 12 - x - 2y\,dA$.

3. Partition the region $R$ into four congruent rectangles with the vertices $(0,0),(0,2),(0,4),(4,0),(8,0),(4,2),(8,2),(4,$ and $(8,4)$.. Use this partition to approximate the signed volume represented by $\iint\limits_R 12 - x - 2y\,dA$ with Riemann sums.

4. Draw a "rectangular" approximation to illustrate your work from the previous problem.

5. Is the double integral $\iint\limits_R f(x,y)dA$ positive? Explain your reasoning.

We will explore a graphical interpretation of iterated integrals using the surface $g(x,y) = 16 - x^2 - y^2$ and the region $R = [0,3] \times [0,2]$.

6. By holding $x$ fixed to one value, we can calculate a slice of area $A(x)$ bounded by $g(x,y)$ and $R$ along this $x$ value. Let $x = 0$, and calculate the area bounded by $g(0,y)$ and $[0,2]$.

$$A(0) = \int_0^2 g(0,y)\,dy$$

7. Draw the area that you calculated in the previous problem, and then give it a thickness of $\Delta x$ to make it a slab of volume $A(0)\Delta x$.

8. Approximate the volume bounded by $g(x,y)$ and $R$ using four volume slabs with a base area $A(x)$ and equal thickness $\Delta x = \frac{3}{4}$.

$$\sum_{i=1}^{4} A(x_i)\Delta x =$$

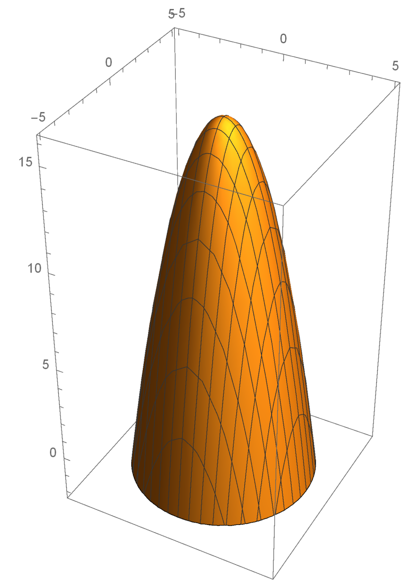
9. To get the exact volume, we need the sum of an infinite number of volume slabs with a base area $A(x)$ and an infinitesimal thickness $dx$ for $x$ from $[0,3]$. Write an integral to represent this volume.

10. Explain why the double integral

$$\int_0^2 \int_0^3 16 - x^2 - y^2 \, dx \, dy$$

will also determine the exact volume bounded by the surface $g(x, y)$ and region $R$, and draw a picture to represent it.

11. Shade the solid whose volume is given by the integral $\int_0^4 \int_0^4 16 - x^2 - y^2 \, dx \, dy$.




12. Now write a double integral to represent the volume of the solid bounded by the function $f(x, y) = 16 - x^2 - y^2$ and the $xy$-plane located in the first octant.

13. Evaluate your integral enough to identify what techniques of integration you need to use to solve.

14. Is there another coordinate system that may result in simpler integration?