Prior projects: Prior to doing this project, students should have done these project:

- Introduction to Mathematica and Graping Project
- Composition of Functions Activity
- What is the Derivative of This Thing Activity
- Chain Rule Project

Philosophy behind this project:
This project serves as motivation for Lagrange multipliers. The proof of Lagrange multipliers is within the reach of students with some scaffolding. In this project, we connect the ideas of parametric equations, composition of functions, chain rule, and optimization. Using these ideas students fill in the blank for the proof for Lagrange multipliers. This project allows for time to discuss proof structure.

The project is also brief enough to take time to discuss the difference in solving systems of equations with Lagrange multipliers compared to their previous experiences with systems of equations.

Learning Goals:

1. Review of parametric equations
2. Review of optimization from Calculus 1
3. Review of composition of functions
4. Review of proof of Lagrange multipliers
5. Review of system of equations

Implementation Notes:

1. It is worth reminding students that one of the most common problems in calculus is finding the extrema of functions. Often optimization is not as simple as just minimizing or maximizing a function, and that there are generally some type of constraint or constraints involved such as "How do we maximize profits given we only have M dollars to invest?"

2. Page 1 has students consider what happens when a surface is constrained by an equation.

3. Page 2 goes through solving the optimization problem using the ideas of parameterization and composition to reduce the problem to a Calculus 1 optimization problem. Note that students will be asked to use Mathematica to draw the space curve they are optimizing. It is worth pointing out to students that they are finding extrema over a curve, not a surface.

4. Page 3 outlines the proof for Lagrange multipliers.
   (a) For many students they are unfamiliar with the structure of proofs. This provides an opportunity to take time to discuss that their arguments must begin with what was given (hypothesis) and build toward the conclusion.
   (b) It is worth mentioning that the creation of the λ variable allows us to compare the directions of the gradients without having to care about their magnitudes.

5. Page 4 has the students revisit the optimization problem using Lagrange multipliers.
   (a) The method for solving systems of equations for Lagrange multipliers is a bit different from what students have worked with previously. It is worth emphasizing that multiple possible options can come from a single equation, such as $4x\lambda = 2x$ gives either $\lambda = \frac{1}{2}$ or $x = 0$ as possible solutions.
   (b) Additionally, students might notice that the Lagrange Multipliers outlined at the top of the page allows for n dimensions. It can be pointed out that this would result in $n+1$ equations and $n+1$ unknowns. n of the unknowns are the coordinates $(x_1, \ldots, x_n)$ and the other unknown is λ.
   (c) Students often forget the last equation that comes from the constraint equation.

6. An extension for students is to have students consider Lagrange multipliers with multiple constraints. The extrema solutions must exist along curve where the two constraints intersect. The normal vectors for both constraints are perpendicular to the intersection curve. If the normal vectors are linearly independent, then any vector perpendicular to the intersection curve can be written as a linear combination of the two normal vectors. If the normal vectors are not linearly independent, then the constraints may provide a solution.

Wrap-Up:

1. While the problem in this project is not too bad to find critical points and plugging them into the function, there are plenty of times where finding potential optimal points along a curve can be pretty messy. Emphasize Lagrange multipliers provide a simpler method to solve constrained optimization problems.

2. It should be important to have students discuss strategies for solving systems of equations for Lagrange multipliers.