Prior projects: Prior to doing this project, students should have done the following projects:

- Parametric Matching
- Parameterization of curves and surfaces

Background content: Prior to doing this project, students should have a working knowledge of the following:

- Parameterization of curves
- Calculation of arclength of a parameterized curve.
- The differential $ds = \sqrt{(x'(t))^2 + (y'(t))^2}\,\Delta t$ represents the length of a short segment of a parameterized curve, traversed over a time interval of length Δt.

Philosophy behind this project:
This project is designed to be done after the students have learned parameterization, and used parameterization to find arclength, but long before they are scheduled to learn line integrals in the textbook (which takes place in Chapter 13). The idea is to front-load some of the harder concepts which in Calc 3 typically get delayed too long. Finding the line integral over a scalar field is presented as a generalization of arclength, replacing a "1" in the integrand with a scalar function.

Learning Goals:

1. Review of calculating arclength of a parameterized curve.
2. Interpretation of arclength as numerically equal to the area of a curved sheet with constant height "1".
3. A line integral is the limit of a sum of rectangular regions. The rectangular regions have a height given by a scalar function and a width of ds, which is the length of a short diagonal segment of a curve.
4. Interpret a line integral as the area of a curved fence, and be able to manipulate this image to a top view and a flattened view.

Implementation Notes:

1. In Problem 1, it is recommended to ask them to draw in the three coordinate axes and label them x, y and z. The axes are labelled in a non-standard way on the sides of the box, and turning this into labels at the tips of the arrows is a good spatial visualization exercise. Mental reorientation of the drawing in Problem 2 is another spatial visualization exercise, as is completion of the graphs in Problem 5.
2. They will need direction about what we are looking for in problems 3 and 4. This direction might look like "Remember when we studied applications of integration in Calc 2? This is the same deal. We are cutting the fence up into slices, finding the area of each rectangular slice, and adding these areas up." If they still are unclear about what to put in the blanks, model the first one for them at the board.

3. This is a good opportunity to review the distinction between Δt and dt. The answers to problems 3 and 4 can be done either way (using either Σ or $\int$ for the summation), but it might be worth reviewing that taking a limit as the width of the rectangles gets small is the bridge between the two.

4. There are some options about how you deal with the differentials in the radical, and students might benefit from seeing this. One option is

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2 (\Delta t)^2} = \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2}\Delta t,$$

which in the limit becomes

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}dt = \sqrt{(x'(t))^2 + (y'(t))^2}dt$$

5. You may choose to introduce the notation Δs and ds for the length of the diagonal segment in Problems 3 and 4. The project doesn't require this (though it is introduced at the very end).

6. The wrap-up for the project is summarized in the box. There are no blanks to fill in, so it's important to go over this in class, so the students absorb the punch-line.

7. If there is time, you can introduce an example of a line integral in 3D. For example, integrate a function of three variables over a helix (quick Calc 2 review of choosing a technique of integration while you're at it?) The line integral of a scalar function of a space curve can be used to find the mass of a wire of varying density.

8. Manipulatives: You can make a fence like in the examples using a paper towel and a pipe cleaner. These can be "straightened out", which is the equivalent of parameterizing to turn it into an ordinary single integral. This process is shown in the last drawing of example 5. It sure would be nice to have some 3D printed models of these objects.