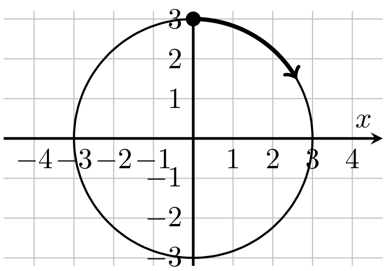
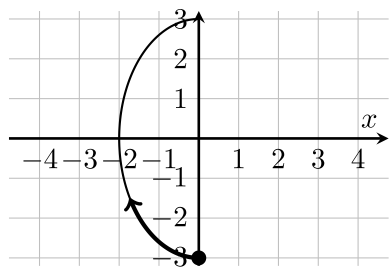
1. We begin by reviewing standard examples of parameterizing curves in the plane and curves in space. This is a skill you will need and return to throughout the semester. Find a parameterization for each of the following curves.



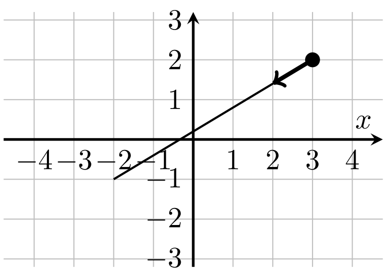
$\begin{cases} x(t) = \underline{3\sin 2t} \\ y(t) = \underline{3\cos 2t} \end{cases}$

$\underline{0} \le t \le \pi$



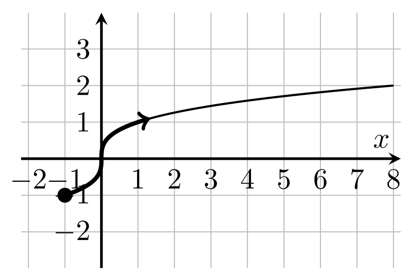
$\begin{cases} x(t) = \underline{-2\sin t} \\ y(t) = \underline{-3\cos t} \end{cases}$

$\underline{0} \le t \le \underline{\pi}$



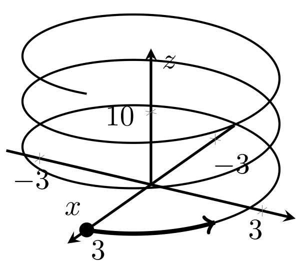
$\begin{cases} x(t) = \underline{3-5t} \\ y(t) = \underline{2-3t} \end{cases}$

$\underline{0} \le t \le \underline{1}$



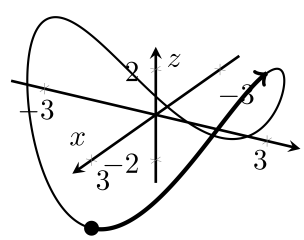
$\begin{cases} x(t) = \underline{t} \\ y(t) = \underline{\sqrt[3]{t}} \end{cases}$

$\underline{-1} \le t \le \underline{8}$



$\begin{cases} x(t) = \underline{3\cos t} \\ y(t) = \underline{3\sin t} \\ z(t) = \underline{t} \end{cases}$

$\underline{0} \le t \le \underline{6\pi}$



$\begin{cases} x(t) = \underline{3\cos t} \\ y(t) = \underline{3\sin t} \\ z(t) = \underline{3\cos 2t} \end{cases}$

$\underline{0} \le t \le \underline{2\pi}$

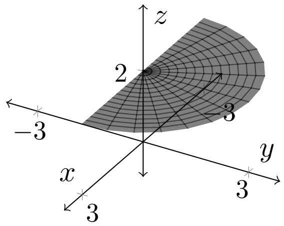
2. Now we will learn how to parameterize surfaces. Here's the idea:

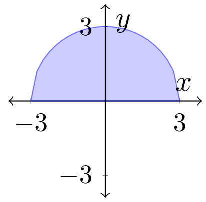
You already know that a curve in the xy-plane can be parameterized by two functions $x(t)$ and $y(t)$, along with a domain for the parameter t. And you already know that a curve in space can be parameterized by three functions $x(t)$, $y(t)$, $z(t)$, along with a domain for the parameter t. As your parameter t varies over the domain, every point on the curve is traced. A curve lying in the plane or in space is essentially one-dimensional, since you can think of it as a deformed line. This is why we use only one parameter t to trace a curve.

On the other hand, a surface is essentially two-dimensional, since you can think of it as a deformed plane. When we parameterize a surface, we need to trace each point in essentially two dimensions. So to fill up the surface, we need to use **two** parameters. We often call these parameters u and v. So to parameterize a surface we need three functions $x(u,v)$, $y(u,v)$ and $z(u,v)$, along with domains for the two parameters u and v. If you are parameterizing using polar or spherical coordinates, it is common to use any of r, θ, ρ or ϕ as the parameters instead of u and v.

On the next page, find parameterizations for each of the surfaces shown. Where it is helpful, you are asked to draw a "top-view" of the surface.

(a) The surface shown below is a half-disk of radius 3 lying at a height of $z = 2$. This is most easily parameterized using polar coordinates, so we will call the parameters r and θ.



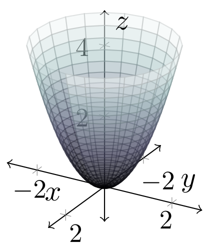


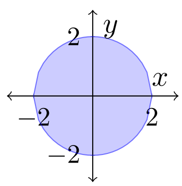
$$\begin{cases} x(r,\theta) = \underline{\ r\cos\theta\ } \\ y(r,\theta) = \underline{\ r\sin\theta\ } \\ z(r,\theta) = \underline{\ 2\ } \end{cases}$$

$\underline{\ 0\ } \le r \le \underline{\ 3\ }$

$\underline{\ 0\ } \le \theta \le \underline{\ \pi\ }$

(b) The surface below is a paraboloid of revolution. The top-view is a disk of radius 2.



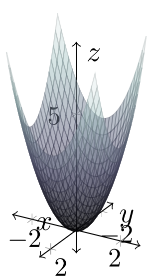


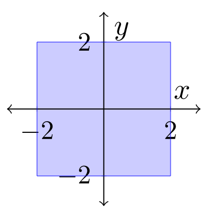
$$\begin{cases} x(r,\theta) = \underline{\ r\cos\theta\ } \\ y(r,\theta) = \underline{\ r\sin\theta\ } \\ z(r,\theta) = \underline{\ r^2\ } \end{cases}$$

$\underline{\ 0\ } \le r \le \underline{\ 2\ }$

$\underline{\ 0\ } \le \theta \le \underline{\ 2\pi\ }$

(c) The surface below is a paraboloid of revolution. This time the variables x and y are each restricted to between -2 and 2. Hint: much like the cube root example on the previous page, in which you knew the formula for the function, just let $x = u$ and $y = v$.



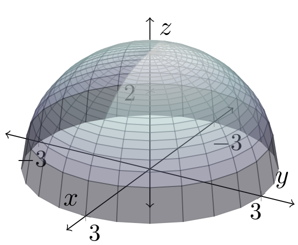


$$\begin{cases} x(u,v) = \underline{\ u\ } \\ y(u,v) = \underline{\ v\ } \\ z(u,v) = \underline{\ u^2 + v^2\ } \end{cases}$$

$\underline{\ -2\ } \le u \le \underline{\ 2\ }$

$\underline{\ -2\ } \le v \le \underline{\ 2\ }$

(d) The surface below is a hemisphere of radius 3. Parameterize it in two different ways, once using cylindrical coordinates and once using spherical coordinates.



$$\begin{cases} x(r,\theta) = \underline{\ r\cos\theta\ } \\ y(r,\theta) = \underline{\ r\sin\theta\ } \\ z(r,\theta) = \underline{\ \sqrt{9-r^2}\ } \end{cases}$$

$\underline{\ 0\ } \le r \le \underline{\ 3\ }$

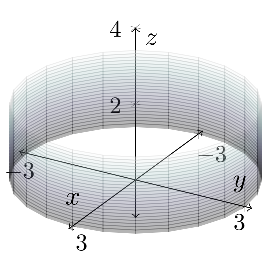
$\underline{\ 0\ } \le \theta \le \underline{\ 2\pi\ }$

$$\begin{cases} x(\theta,\phi) = \underline{\ 3\sin\phi\cos\theta\ } \\ y(\theta,\phi) = \underline{\ 3\sin\phi\sin\theta\ } \\ z(\theta,\phi) = \underline{\ 3\cos\phi\ } \end{cases}$$

$\underline{\ 0\ } \le \theta \le \underline{\ 2\pi\ }$

$\underline{\ 0\ } \le \phi \le \underline{\ \frac{\pi}{2}\ }$

(e) The surface below is a cylinder of radius 3 and height 2.



$$\begin{cases} x(\underline{\ \theta\ }, \underline{\ z\ }) = \underline{\ 3\cos\theta\ } \\ y(\underline{\ \theta\ }, \underline{\ z\ }) = \underline{\ 3\sin\theta\ } \\ z(\underline{\ \theta\ }, \underline{\ z\ }) = \underline{\ z\ } \end{cases}$$

$\underline{\ 0\ } \le \underline{\ \theta\ } \le \underline{\ 2\pi\ }$

$\underline{\ 0\ } \le \underline{\ z\ } \le \underline{\ 2\ }$