Background content: Prior to doing this project, students should have a working knowledge of the following:

- Ability to distinguish between scalars and vectors in a wide variety of situations.
- Dot product of two 3D vectors and its relationship to the cosine of the angle between the vectors.
- Cross-product of two 3D vectors, and the geometric interpretation (a vector which points perpendicular to both starting vectors, and whose magnitude is the area of the parallelogram having the two starting vectors as adjacent sides.)
- Scalar triple product, and the geometric interpretation as the volume of a parallelopiped spanned by the three vectors.
- Equations of lines in 3-space (parametric form, vector form, symmetric form).
- Equations of planes in 3-space.
- Ability to distinguish between scalar functions and vector functions, and to determine the dimension of the domain and co-domain by looking at a formula.

Philosophy behind this project:
This project is designed to address the significant problem we notice in Calc 3: that students look at mathematical structures from the course and, without even stopping to ask themselves "What is this thing?", they begin computing and calculating in nonsense ways. The goal of the project is for them to practice looking at combinations of some of the objects they studied, determine whether they are scalars or vectors, and interpret their meanings. Mixed in with these examples are some that don't even make sense (for example, the cross-product of a scalar and a vector, composition of two functions in which the co-domains and domains don't match properly), so the students are forced to think carefully.

Learning Goals:

1. When confronted with any combination of scalars, vectors, dot- or cross-products, equations or scalar- or vector-valued functions, students will stop to ask themselves "What is this thing?"
2. Students will be able to recognize when objects they have met in the course are combined in ways that don't make sense.
3. Students will review the geometric interpretations of dot product, cross-product and scalar triple product.
4. Students will review the vector form and symmetric form of equations of lines in 3-space. In particular, they will know how to find the direction of a line from either the vector form or symmetric form of the equation.
5. Students will recognize a cross-product from its form as a determinant.
6. Students will recognize in context that the cross-product of two vectors is perpendicular to each of them.

7. Students will review how to determine the normal of a plane given its equation.
8. Students will review how to calculate the equation of the intersection of two planes.
9. Students will determine when two functions can be composed.

Implementation Notes:

1. Some of the questions are open to interpretation. We are more concerned that students are actively discussing and debating their interpretation with valid arguments than whether they agree with our interpretation.
2. This project will be very difficult for most of the students. TAs will probably need to collect the attention of the class often and give short bits of direct instruction. Capitalize on as many teaching moments as possible.
3. Number 3: the issue here is whether or not we are willing to interpret the dot as scalar multiplication, or if we think it must be a dot product. This is the question they should be realizing to ask themselves. If students think the dot must represent a dot product, then Number 3 makes no sense. If they realize that it could represent scalar multiplication, then c and f are possible answers.
4. Number 5: A question to ask the students: why is this not a scalar triple product? Could we modify it to make it a scalar triple product?
5. Numbers 6 (and 7 and 13), we are identifying the functions themselves as lines. Technically these are functions whose graph is a line. We think this distinction is beyond the scope of this course, so we accept j, m (and k) as correct answers.
6. Number 8: This is perhaps the most difficult of the questions. One solution is to take the cross-product of the two normal vectors to get the direction of the line (telling you that j is an option), then substitute points to eliminate choice k and confirm choice m. Another way to confirm choice m is to find two points on the line, and substitute to show both points lie on both planes. Then choice j can be selected by confirming its direction is the same as the direction of the line in choice m.
7. Students might not recognize this as the symmetric form of the equation of a line. Everything there looks pretty linear though, so either a line or a plane should be good intuitive choices for them. You could suggest that they turn it into a parameterized curve by letting each expression equal t. Solving then gives $x = t + 2$, $y = 3t - 1$ and $z = 4t + 3$. At this point they should recognize that it is a line, and be able to detremine the direction.
8. Number 13 just requires them to combine components from Number 7 to get an alternate vector representation of the same line.
9. For Numbers 14 and 15, they'll benefit from a short explanation of how to use mapping diagrams (such as $\mathbb{R}^2 \xmapsto{f} \mathbb{R} \xmapsto{\mathbf{g}} \mathbb{R}^3$) to keep track of compositions.
10. Number 14 does not make sense because it is clearly indicated that f is a function of just two variables. However, 17 and 18 are open to two interpretations - they can be viewed as either a curve in a plane or a surface in space.