Background content: Students should recognize all types of integrals from three semesters of calculus, the associated notation, and interpretation of these integrals in applications. The types of integrals include: single integrals, double integrals, triple integrals, integrals for arclength, line integrals over scalar fields, line integrals over vector fields, integrals for surface area, surface integrals over scalar fields, and surface integrals over vector fields.

Philosophy behind this project:
This project is the culmination of our series of “What is this thing?” projects. It is a review exercise covering all types of integrals from calculus, asking the students to recognize the notation for each type of integral, interpret the meaning of the integral, and associate the integral with an application. We find that in this course students get overwhelmed by the variety of types of integrals and their purposes. Only with a firm grasp on the types of integrals and their interpretations is intuitive understanding of the Fundamental Theorems possible.

Learning Goals:

1. Create a mental outline of the types of integrals and their applications.
2. When presented with an integral, be able to state what type of integral it is and what it is typically used for.
3. When presented with an application, be able to state what type of integral to use.
4. Explain the difference between a line integral over a scalar field and a line integral over a vector field. Explain the difference between a surface integral over a scalar field and a surface integral over a vector field.
5. Fluidly recognize the variety of notations used for different types of integrals (e.g., $\displaystyle\int_C \mathbf{F}\cdot\mathbf{T}\,ds = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt$.)

Implementation Notes: There are 18 integral cards to match with 18 application cards. Students are expected to know both general form and parameterized form and the variety of notation.

1. It might be a valuable mini-lecture to: review the difference between an unparameterized versus parameterized form of an integral. Also, show how to use substitutions to derive the parameterized version from the general version. Since parameterization by arclength is typically difficult for them, it will be interesting for them to see what happens to the differential ds in line integrals (and the dS in surface integrals)
2. There are 6 cards with a“1” as an integrand. Here are some questions to ask students about each of these: When you replace the “1” in the integrand with an arbitrary function, what type of function should it be? When you replace the “1” with an arbitrary function, how does that change what the integral is calculating? In each case, explain why this makes sense. You can also go the other way, replacing a scalar function in the integrand with the constant “1”. This exercise should help them with understanding the geometric meaning of each of the differentials.
3. Below are some cards that were in the original draft, but didn’t make the final cut. You can use these in your wrap-up:
$\int_a^b \int_{c(x)}^{d(x)} 1\,dy\,dx$ (Iterated double integral which gives an area)

$\int_a^b \int_{c(x)}^{d(x)} f(x, y)\, dy\, dx$ (Iterated double integral which gives a volume)
$\int_C \mathbf{F} \cdot d\mathbf{r}$ (Line integral of a vector field which gives work done)
$\iint_S \mathbf{F} \cdot d\mathbf{S}$ (Surface integral of a vector field which gives flux)
$\iiint_E f\, dz\, dy\, dx$ (Iterated triple integral which gives the mass of a 3D solid)

4. Here are some questions to ask students:
   - What is the difference between a line integral (or surface integral) over a scalar field and a line integral (or surface integral) over a vector field?
   - If I'm looking at a line integral (or surface integral), how do I tell if it is an integral over a scalar field or a vector field?
   - Read one of the application cards to them and ask them to write down the integral that goes with it.
   - Explain how a volume can be represented by either a double or a triple integral. Similarly, how can an area be represented by either a single integral or a double integral?