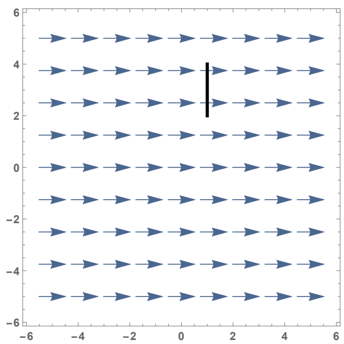
When studying surface integrals over vector fields we often use the word flux. To develop our understanding of flux, we will consider the more intuitive definition of flux as the *total rate of flow across a boundary.*

Suppose we cover the plane with water that is flowing to the right at the rate of 4 ft/sec. We can represent this flowing pool as a vector field **F**. We can use our knowledge of vectors to compute the flux across the line segment from $(1,2)$ to $(1,4)$. First, we visualize the flowing pool as a vector field in 2-space.



1. Find and draw **unit** normal vector **n** to the line segment. Choose the direction pointing along the flow. *Notice the unit normal is unitless.*

2. Write the flow rate of the water as a vector **F**. Include appropriate units.

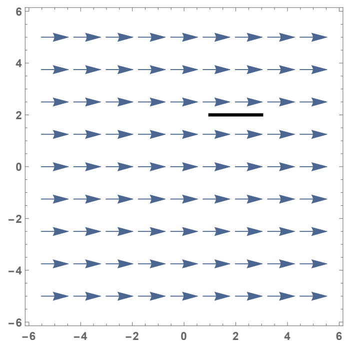
3. To compute the amount of flow in the direction of the unit normal vector at each point, find the projection of the flow rate vector **F** onto the unit normal vector **n**. Include appropriate units.

To calculate the flux across a line segment we can use the following idea:

$$\text{Flux}=\text{Flow Rate}\times\text{Length of Segment}$$

4. Now we multiply the result of this dot product by the length of the line segment to calculate the total rate of water flow across each point of the line segment. Include appropriate units.

5. Consider the flux across the line segment from $(1, 2)$ to $(3, 2)$.



   (a) Use vectors and following the process in the previous page to compute the flux across the line segment. Include units.

   $\mathbf{F} =$

   $\mathbf{n} =$

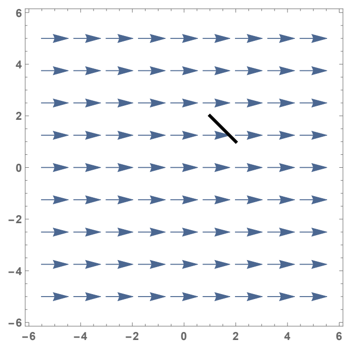
   $\mathbf{F} \cdot \mathbf{n} =$

   Segment Length $=$

   Total Flux $=$

   (b) Explain why your answer to part (a) makes sense intuitively.

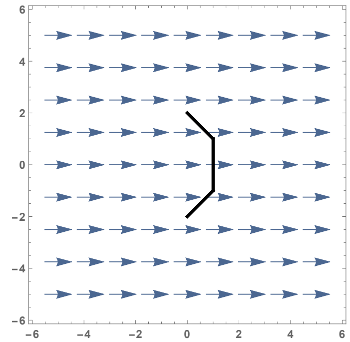
6. Now consider the flux across the line segment from $(1, 2)$ to $(2, 1)$.



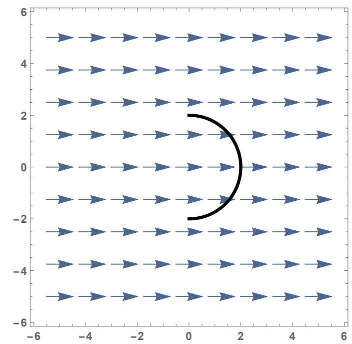
   (a) Use vectors to compute the flux across the line segment. Include units.

7. Consider the flux across the curve C consisting of line segments from $(0,2)$ to $(1,1)$, from $(1,1)$ to $(1,-1)$, and from $(1,-1)$ to $(0,-2)$.

    (a) Use vectors to compute the flux across the curve C.



8. How might we calculate the flux across the curve C, defined by $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t)\rangle$ with domain of $t \in [\frac{-\pi}{2}, \frac{\pi}{2}]$ in our flowing pool of water?



To help think about this problem, consider the following problems.

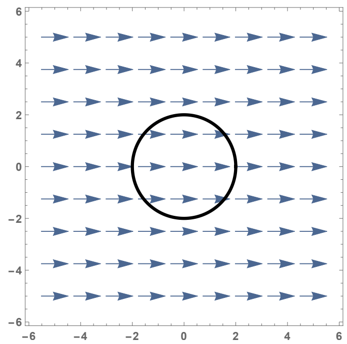
(a) Find the unit normal vector to the curve for all points along the curve. *Hint: The unit normal vector is perpendicular to the unit tangent vector.*

(b) To compute the amount of flow in the direction of the unit normal vector at one point, take the dot product of your unit normal vector and the flow vector.

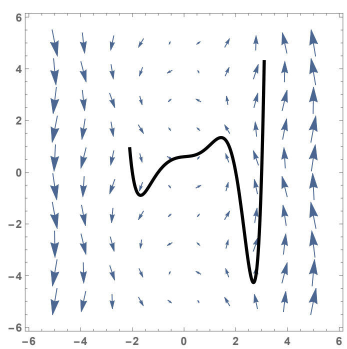
(c) How can we multiply the result of this dot product by an infinitesimal length of the curve to find the rate of flow across a piece of the curve?

(d) Find the total flow rate across the curve.

9. Calculate the flux across the closed curve $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t)\rangle$ with domain of $t \in [0, 2\pi]$ in our flowing pool of water? *Note: We can also use a vector form of Green's Theorem to calculate this.*



Now we will generalize this idea for any continuous, smooth curve along a vector field, such as:



> In general, given any continuous, smooth curve C along a vector field $\mathbf{F}$ we can calculate the flux, or total rate of flow, across the curve by evaluating the integral
>
> $$\int_C \underline{\hspace{6cm}}$$

10. We now extend this idea to higher dimensions. If we now move from our vector field $\mathbf{F}$ in 2-space to the vector field $\mathbf{G}$ in 3-space, the analogous to our curve C in 2-space is a ________________________ in 3-space. How might we compute the flux across this boundary in 3-space?