Background content: Prior to doing this project, students should be able to do the following:

- Use the dot product to find scalar projection of one vector onto another.
- Compute unit vectors.
- Find derivatives of parameterized curves.
- Use arc length to find the total length of various curves.
- Understand parameterization of curves.
- Apply integration of polar coordinates

Philosophy behind this project:
The textbook uses the surface integral of a vector field over a solid to define the idea of flux, and during this point of the semester we find that students can work through the integration without understanding what the result of their integration represents or means. The idea behind this project is to build students' intuitive understanding of flux and surface integrals of vector fields by examining the concept of flux in $\mathbb{R}^2$. The aim is to help provide meaning to surface integrals of vector fields.

Learning Goals:

1. Review and apply concepts of unit vector, dot product, scalar projection, orthogonality, arc length in context of flux.
2. Develop understanding of flux as the total flow rate across a boundary.
3. Solve the total rate of flow across various curve boundaries.
4. Generalize their work to calculate flux across any continuous, smooth curves along a vector field.
5. Relate flux across a curve boundary to flux across a surface boundary.
6. Recognize the normal vector of a curve is orthogonal to its tangent vector.
7. Connect Green's Theorem to the concept of flux across a curve boundary.

Implementation Notes:

1. During the project students consider the total rate of flow of water across a curve boundary, so they are having to think how much flow is crossing each point of a curve. This can be connected to the concept of calculating the total work done as we move along a path over some force field, since this has students think about the amount of work is required to move along each point of a path. We prep students for this project by discussing that we are going to tweak the concept of line integrals over vector fields.

2. A point of emphasis in the concept of flux and surface integrals of vector fields is the orientation of a given surface. Since this project works primarily in two space, we tell students the orientation of the normal vector to be in the same direction as the flow of water. This is worth mentioning throughout the project, and in particular Problem 8.

3. Page 1 sets up the idea for the entire project, so we want to ensure that students understand the overall idea. There are a few concepts that students will need to recall as they work through the page. It may be worth stopping for a whole class discussion on scalar projection since it is one of the two foundational concepts of flux. The other foundational concept is calculating the length (or area) of the boundary.

4. Page 2 is an extension of Page 1 where students find the flux across straight lines (horizontal and at an angle). For Problem 5, be sure that there is a discussion on why the flux across the horizontal line boundary is zero. Students will think that water is going over the line, but note that there is no water that crosses from one side of the boundary to the other.

5. Page 3 scaffolds students from working with boundaries that have constant normal vectors on Page 1 and 2 to a boundary with a variable normal vector.

6. Page 4 is intended to have students recall previous knowledge (tangent vectors, arc length, and path independence), while connecting to the new idea of flux. There is a chance for confusion in Problem 8(a) in dealing with perpendicular vectors and orientating their normal vector. If students choose the wrong orientation, have them work through 8(b) and recognize that they will be getting negative values for their flow. This is also a chance to discuss the importance of orientation for their normal vectors. When students answer 8(d), direct their attention that the flux for the segmented "half-circle" and smooth half-circle are equal. This provides a chance to revisit the idea of path independence.

7. Page 5 connects the work they've been doing to the vector form of Green's Theorem, flux across a general curve, and then extends the idea of flux to surface integrals.

Wrap-Up:

1. The wrap-up should focus on reviewing what flux represents, and how they can use this understanding for thinking about how to calculate flux.