Prior projects: Prior to doing this project, students should have done these project:

- Introduction to Introduction to Line Integrals
- Line Integrals over Vector Fields

Philosophy behind this project:
We often begin covering vector calculus in the last third of the semester. Students become overwhelmed with all the new concepts that are packed into this time, and this results in them focusing on the computational aspects of the material instead. To address this, the project is completely conceptual. The goal is to have students develop intuition and understanding of what their computational work represents.

Learning Goals:

1. Identify types of integration based on differentials: $ds, dV, dt, d\mathbf{r}, dA, dx, \partial x, Df, dr$, and $d\vec{r}$.
2. Review of the various forms of integration covered so far in the their study of calculus:
   - over one dimensional domain
   - integrate with respect to arc length
   - line integral over vector field
   - over two dimensional domain
   - over three dimensional domain
3. Differentiate between line integrals over scalar fields and vector fields based of integral representation
4. Understand different visual representations of line integrals over scalar fields:
   - 2D area representation
   - Graph of 2D scalar field and curve C
   - A 3D representation of a scalar field and path along scalar field above C
5. Understand different visual representations of line integrals over vector fields:
   - A 2D area representation
   - A graph of 2D vector field and curve C.

Implementation Notes:

1. Problem 1 should go quickly, but it is worth making sure that students explain their reasoning for their answers - these discussions can be contained within the small groups. Note that some of parts have more than one differential.

2. Problem 2 connects different representations of line integrals to what they can conceptually represent. This is a good chance to have a whole class discussion about the reasoning behind the different line integrals representations (i.e. Why can line integrals over vector fields represent work? Why can line integrals over scalar fields represent an area of a curved fence or the mass of a curved wire?)

3. The idea for Page 2 and Page 3 came from two animations on the Wikipedia page for line integrals. The images for 3(a) and 4(a) come directly from the animations.

4. Let students attempt to grasp what is happening with the visual representations on Page 2 and 3. If their struggles start building toward an unproductive frustration, then direct them to Wikipedia page to watch the animations.

5. Problem 3(a) has students identify specific parts of the 2D area representation of line integral over scalar field to both its representation of 2D scalar field and 3D representation of scalar field. Be sure students recognize the starting and ending point of the curve C in the graph of the 2D scalar field.

6. Problem 3(b) reverses the idea from part (a). Now students must construct the 2D area representation of the line integral over scalar field. Just like in part (a), be sure students recognize the starting and ending points of the curve. There should be a discussion about what crossing the axis on their area representation connects to in the other visuals.

7. Problem 4(a) is similar to 3(a), where students identify specific parts of the 2D area representation in the graph of a 2D vector field. Be sure to question students' reasoning for their placement of X's and 0's.

8. 4(b) is similar to 3(b), where students construct 2D area representations for line integrals over vector fields. The 2D area representation starts with a negative with a small dip down, before heading toward the positives. Have students discuss why there should be this small dip (magnitude of vectors in opposite direction of path increase). Also, make sure that students identify where the area representation crosses the axis.

Wrap-Up:

1. The wrap-up should be a summary discussion about the difference between line integrals over scalar fields and vector fields. Included in this discussion should be what the two integrals can represent and why those representations are appropriate.