Background content: Prior to doing this project, students should be able to do the following:

- Parameterize surfaces in cylindrical and spherical coordinates.
- Find intervals to restrict the parameters to, in order to define a specified bounded surface.
- Use a parameterization to find the surface area of a bounded surface.

Philosophy behind this project:
We present surface integrals over scalar fields prior to its placement in the textbook. The final chapter in the textbook presents many types of integrals in quick succession, not leaving enough time for students to internalize the meanings and calculation of the types of integrals before learning the Fundamental Theorems. Our early placement of surface integrals over scalar fields is an attempt to mitigate this problem. Surface integrals over scalar fields are presented as a generalization of surface areas.

Learning Goals:

1. Review parameterization in cylindrical and spherical coordinates.
2. Review the process for finding surface area of a bounded surface.
3. Be able to calculate the surface area of a specified surface.
4. Be able to state a geometric interpretation for the differential in a surface area integral. In other words, be able to clearly state that $\Delta S = |\mathbf{r}_r \times \mathbf{r}_\theta|\,\Delta r\,\Delta\theta$ represents the area of the tiny paralellogram which the parameterization maps a Δu-by-Δv rectangle onto.
5. See the surface area integral $\iint_R |\mathbf{r}_u \times \mathbf{r}_v|\,du\,dv$ as equivalent to $\iint_R 1\,|\mathbf{r}_u \times \mathbf{r}_v|\,du\,dv$, and realize the "1" in the integrand can be generalized to an arbitrary function $f(x, y, z)$.
6. Recognize that the function $f(x, y, z)$ in the integrand of a surface integral is a distinct entity and has a distinct interpretation from the function defining the surface of integration.
7. Realize that a surface integral can be used to calculate the mass of a curved lamina with variable density.
8. Be able to evaluate evaluate a surface area of a specified function over a specified surface, of reasonable difficulty.
9. Be able to interpret dS as the area of an infinitesimal parallelogram on the surface, and state that $dS = |\mathbf{r}_r \times \mathbf{r}_\theta|\,\Delta r\,\Delta\theta$.

Implementation Notes:

1. The first problem asks students to review the process for finding surface area, which they should have learned a few days prior. The goal of this problem is not merely to review the procedure for finding surface area and to practice calculating it, but more importantly to interpret the parameterization as mapping rectangles in the u-v plane into parallelograms that lie on the surface, and that the area of these parallelograms is calculated by $|\mathbf{r}_r \times \mathbf{r}_\theta|\,\Delta r\,\Delta\theta$. This is the goal both for the fill-in-the-blank in Problem 1, and for Problem 2.

2. Problem 3 is a lead-in to the surface integral in Problem 4. The goal is for them to effectively use communication in groups to resolve their cognitive dissonance. Discussion among the groups should include identifying units (the units on $dS = |\mathbf{r}_r \times \mathbf{r}_\theta|\,\Delta r\,\Delta\theta$ are square-units), and that if we insert a "1" with corresponding units to the integral, $\iint_R 1\text{unit}\,dA$ gives the volume of a solid with height 1 and area $\iint_R dA$. Thus the double integral $\iint_R 1\,dA$ gives an area. Similarly, the double integral $\iint_R 1dS = \iint_R dS = \iint_R |\mathbf{r}_u \times \mathbf{r}_v|\,du\,dv$ gives an area. These ideas discussed in individual groups should be shared with the whole class. Note that this theme of a 1 in the integrand returns in future projects.

3. In Problem 4, we replace the "1" in the surface area integrand with a function of three variables. This is the first time they have seen surface integrals. They must use their work with density and mass from Calc 2 to realize that replacing the integrand with the density function results in an integral that gives the mass of the object.

4. Students are asked to calculate the surface integral in Problem 4 in cylindrical coordinates. They can use the parameterization they found from Problem 1 as a starting point. The solutions to the project demonstrate how to do a "sanity-check" on the numerical result. It is worth asking them in their groups or as a whole-class discussion how to reality-check their results.

5. The framed box on the second page summarizes the notation for and the process of calculating a surface integral. Display a graph such as in the solutions to problem 1 and point out the parallelograms whose area is represented by dS. Clarify that the integral in Problem 1 is adding up the areas of these rectangles, while the integral in Problem 4 is multiplying the area of each these parallelograms by their respective densities, then summing these masses. Review again that we calculate the area of each parallelogram by $dS = |\mathbf{r}_r \times \mathbf{r}_\theta|\,\Delta r\,\Delta\theta$.

6. The add-on questions for at-home practice are important. They give students a chance to practice calculating surface integrals and to review spherical coordinates. Students who have a firm grasp on calculating in both cylindrical and spherical coordinates fare better in the final chapter of the semester.