Answer the following questions about the "Gradient graphically" project:

1. Confirm analytically that the gradient of each of formulas for the potential functions matches the formula for the gradient vector field.

2. The level curves for the cone (graph $\heartsuit$) and the paraboloid (graph $\diamondsuit$) are both concentric circles. How did you determine which set of level curves match the cone?

3. The gradient field for the cone and the paraboloid both consist of vectors that all point directly away from the origin. How did you determine which of those two fields match the cone?

4. Look at each gradient vector field. Only by looking at this field, try to imagine the surface of the potential function it represents. Try the same thing by looking at the graphs of the level curves.

5. The gradient vector field on card 3 matches the level curves from Card C, but it looks like no matter how it is oriented, the vectors point perpendicular to the level curves. How do you determine which way to orient it? Do you require the graph of card $\clubsuit$ do confirm this? Explain.

6. Explain how to confirm directly that formula card b, matches potential function graph $\diamondsuit$.

7. Explain how to directly confirm that formula card a matches the level curves graph C.

8. Look at card γ, giving the formula for a vector field. Calculate the magnitude of the gradient vector, then explain how this confirms the match with vector field card 2. Also explain why the vector at each point in the plane is directed exactly away from the origin.

9. Suppose a potential function is shifted upward in the z-direction. For example, $f(x, y) = x^2 + y^2$ becomes $f(x, y) = x^2 + y^2 + 5$. How does this affect the graphs of the level curves, and the gradient vector field?

10. Look at vector field 1 and potential function $\diamondsuit$. A student incorrectly says that these vectors point perpendicular to the surface of the potential function. Explain what is wrong with their thinking, and explain how to find the vectors that point perpendicular to the surface of the potential function itself.