$\mathbf{F}(x, y)=\left\langle\frac{x}{x^{2}+y^{2}}, \frac{y}{x^{2}+y^{2}}\right\rangle$

$\square$
$\mathbf{F}(x, y)=\langle-x, 2\rangle$

$\square$
$\mathbf{F}(x, y)=\langle y, 0\rangle$

$\mathbf{F}(x, y)=\langle x-y, x+y\rangle$


$\mathbf{F}$ is not conservative, because the derivative of its $x$-component with respect to $y$ is 1 and derivative of the $y$-component with respect to $x$ is 0 , and thus they are not the same.
$\mathbf{F}$ is not conservative, because there are closed paths for which $\int_{C} \mathbf{F} \cdot d \mathbf{r}$ is not 0 . For example, the line integral along a counter-clockwise circle of radius 5 centered at the origin is negative.
$\mathbf{F}$ is not conservative because the anti-derivative of $y$ with respect to $x$ is $x y+c(y)$, and the anti-derivative of 0 with respect to $y$ is $k(x)$. These two functions cannot be made equal by the choice of the constants.
$\mathbf{F}$ is conservative, because $\mathbf{F}=\nabla f$, where $f=x y$
$\mathbf{F}$ is conservative, because $\mathbf{F}=\nabla f$,
where $f=-\frac{1}{2} x^{2}+2 y$
$\mathbf{F}$ is not conservative, because $\int_{C} \mathbf{F} \cdot d \mathbf{r}$ is not path-independent. A counter-clockwise semi-circular path from $(0,5)$ to $(0,-5)$ gives a positive value while a clockwise path gives a negative value.

|  |
| ---: |
| The value of $\int_{C} \mathbf{F} \cdot d \mathbf{r}$ |
| for any path from $(5,5)$ to |
| $(0,5)$ is always positive. |
|  |
|  |
| 4 |

$\mathbf{F}$ is not conservative, because $\int_{C} \mathbf{F} \cdot d \mathbf{r}$ is not path-independent. A
counter-clockwise semi-circular path from $(0,5)$ to $(0,-5)$ gives a positive value while a clockwise path gives a negative value.

The value of $\int_{C} \mathbf{F} \cdot d \mathbf{r}$ for any path from $(5,5)$ to $(0,5)$ is always -25 .
$\mathbf{F}$ is not conservative, because the the curl of $\langle x-y, x+y, 0\rangle$ is not 0 .
$\mathbf{F}$ is conservative, because the derivative of its $x$-component with respect to $y$ and the derivative of its $y$-component with respect to $x$ are both 1 , and thus the same.

