Prior projects: Prior to doing this project, students should have done the following projects:

- Vector Field Matching project
- Line integrals over vector fields project

Background content: Prior to doing this project, students should have a working knowledge of the following:

- State the definition of conservative, that a vector field is conservative if it is the gradient of some scalar function (called a potential function)
- Given the formula for a vector field $\mathbf{F}$, test if it is conservative by integrating each component with respect to the appropriate variable and resolving the results by adding functions constant with respect to the appropriate variable.
- Given the formula for a vector field $\mathbf{F}$, determine if it is conservative by computing $\text{curl}\,\mathbf{F}$ and checking if it is 0. In 2-D this means checking that $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$
- State the in a conservative field, line integrals must be independent of path.
- State the in a conservative field, line integrals over a closed curve must be 0.
- State the Fundamental Theorem for Line Integrals.

Philosophy behind this project:
There are many inter-twined concepts relating to conservative fields, and the goal of this project is to facilitate the students making connections between these concepts and creating an organized mental schema of how these concepts relate. The basic ideas we intend that they join, include the FTC for line integrals and required conditions for a field to be conservative. (If the field is to be conservative, then the mixed partials of its potential function will be equal. The components of the field given should already be the first derivatives of the potential function, so taking the opposite derivatives yields what would be the mixed partials, and we check these for equality. In three-dimensions, this is done with the curl.) We also intend them to connect the idea of path-independence, line integrals over closed curves being zero, and conservativity of the vector field.

Learning Goals:

1. Find the potential function for a conservative field by integrating the two component functions and resolving the constants.
2. Realize independently that if the two integrals calculated to find a potential function for a field cannot be resolved, this means that the field must not be conservative.
3. When presented with unequal values for $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$, or a non-zero curl, conclude independently that the field must not be conservative.
4. Faced with the non-path-independence of a line integral, conclude independently that the field must be non-conservative.
5. Faced with a closed curve giving a line integral that is not 0, conclude independently that the field must be non-conservative.

6. Realize independently that in a conservative field, a line integral can be calculated by finding the potential function, and finding its difference evaluated at the endpoints.

7. Determine graphically the sign of a line integral along a directed curve by seeing which direction the arrows point.

Implementation Notes: The first page of the project is a "placemat". The second page is cut into cards that are each placed on the appropriate empty box of the placemat. When a card can be placed on multiple placemats, then there are multiple copies of it in the card list, labelled with the same number values. Thus, there is only one valid solution to the project.

1. Correct answer: Field 'A' has one card congruent to 1 (mod 6). Fields B and F should be congruent to 0 (mod 6). Field C should be congruent to 3 (mod 6). Cards matching field D should be congruent to 4 (mod 6).

2. Cards 1, 3, 9, 15, 4 and 10 apply to fields that are conservative, and the rest apply to fields that are not conservative. One possible strategy is to sort the cards first for whether or not they apply to a conservative field.

3. Note that the descriptions on 5 and 6 follow from path-independence and specific knowledge about the vector field, but neither independently imply the field is conservative.

4. Cards 1, 3 and 10 can be matched by finding ∇f for each potential function.

5. Card 9 hints at path-independence (though it doesn't guarantee it). Possible strategy: look at a variety of paths from the start to the end point to see which paths give a negative value, and give a negative value regardless of path. A better strategy: determine which fields are conservative before doing card 9, so you will have the potential function. The, use the FTC for line integrals to check that the value of the line integral really is -25. In fact, I **must** use the FTC for line integrals to be able to distinguish fields A and C.

6. The cards involving derivatives and integrals (15, 5, 17, 12) differ subtly. In card 17, we attempt (and fail) to find a potential function, while in cards 15, 17 and 12 we are considering equality of $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$. Make sure students note the difference (in the former case we are integrating the x-component with respect to x and the y-component with respect to y, whereas in the latter we are differentiating the x-component with respect to y, and the y-component with respect to x.) The relevant theorems are Theorems 5 and 6 of Section 13.3 and Theorems 3 and 4 from Section 13.5. Note that inequality of $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ guarantees a field is not conservative, but that to use the Theorem to guarantee the field is conservative (Card 15), we need the equality of $\frac{\partial P}{\partial y}$ and $\frac{\partial Q}{\partial x}$ as well as the continuity of those derivatives.

7. Advanced question: If I know that I have path-independence between any two specific points A and B, then do I have path independence between any two points? (Answer: Yes. Extend two paths between points C and D into a line from A to C, then the path from C to D, followed by a line from D to B. The line integral along these two extended paths is the same, and thus the integral along the paths from C to D must also be the same.)