Background content: Prior to doing this project, students should have a working knowledge of the following:

- Determining the dimension of the domain and codomain in a function given by a formula.
- Distinguishing between scalar-valued and vector-valued functions.

Philosophy behind this project:
This project, together with the "What is the derivative of this thing?" project together are precursors to the chain rule project. We have found that when students are confronted with a function in Calc 3, they do not automatically question what the domain and codomain are. In other words, they do not ask themselves "What is this thing?", which poses problems when they try to find the derivative of a composition. We are attempting to remedy this problem by having them practice with considering dimensions of input/output, and determining which compositions are valid.

Learning Goals:

1. When confronted with a formula for a function, determine the dimension of the input/output without being prompted for it.
2. Practice writing these dimensions down in mapping diagrams, either in the format $f : \mathbb{R}^2 \mapsto \mathbb{R}$ or $\mathbb{R}^2 \xmapsto{\mathbf{f}} \mathbb{R}$
3. Determine if a composition is valid (either with or without the aid of a mapping diagram).

Implementation Notes:

1. There is room to draw mapping diagrams on the right of the definition of the functions. We chose not to explicitly ask students to draw mapping diagrams, because we want them to realize independently that it will be helpful to do it. Without doing so, they will may be able to do most of the questions, but the last question will be difficult. Please use your discretion to determine when you should advise them to do so (or find a student who has done so and share their idea with the class).
2. For each of problems 1-8, they should determine whether or not the given composition makes sense. If it does, they should fill in the dimension of the domain and codomain of the composition.
3. There is a subtlety here that we chose to ignore, but be aware in case it comes up: We internally think of g (for example) as taking a point or vector as an input, though we write it (as is customary) as though it takes two individual numbers as an input. On the other hand (as is customary), we explicitly write the output as a vector. In other words, we write $g(x, y)$, not $g(\langle x, y\rangle)$.
4. Question for students: Explain why sometimes a composition in one order works while the other order does not.
5. Question for students: For two functions, in what circumstances would both orders of composition be valid?