Worksheet on Truth Tables and Boolean Algebra

September 24, 2015

1. Write a truth table for the logical statement $\neg(P \vee Q) \implies (\neg P \wedge Q)$. Do it step by step (i.e. include columns for $\neg P$ and $P \vee Q$ etc., building up to the full expression). Note that the symbol $\neg$ has the same meaning as $\sim$ in your book, and these two symbols both mean 'not'.

2. Is the logical statement above true? (Hint: Is $P \wedge Q$ or $P \vee Q$ true?)

3. The logical statement above is logically equivalent to one of the 'basic' statements, $\neg P$, $P \wedge Q$, etc. Which one?

4. Prove DeMorgan's Laws:
   - $\neg(P \wedge Q) = (\neg P) \vee (\neg Q)$,
   - $\neg(P \vee Q) = (\neg P) \wedge (\neg Q)$.

5. A *tautology* is a Boolean expression that evaluates to TRUE for all possible values of its variables. Work together (as always) to come up with an example of a tautology in two variables (you might try one variable first if you are stuck). Provide a proof (that is, a truth table) that it is a tautology.

6. A *contradiction* is a Boolean expression that evaluates to FALSE for all possible values of its variables. Come up with an example of a contradiction in two variables and prove that it is one.

7. How many lines (besides the header) does the truth table for a Boolean expression in 13 variables have?

8. How many logically distinct Boolean operations could you define on two variables? Writing out all possibilities is possible but a hassle. Instead, do this by being clever, and thinking about counting problems.

9. How many logically distinct Boolean operations could you define in n variables?

10. Show that $P \Leftrightarrow Q$ is logically equivalent to $(P \implies Q) \wedge (Q \implies P)$. Thus, in some sense $\Leftrightarrow$ isn't needed – it can be 'generated' by $\implies$ and $\wedge$.

11. Find a way to generate $P \implies Q$ using only $\vee$, $\wedge$ and $\neg$.

12. With reference to the last few questions, how many Boolean operations are needed to generate all possible Boolean operations? Don't do this by exhaustive search or blind messing about. But don't be afraid to experiment either. The point is to do some experimentation and look for patterns, then prove relevant facts, or partial statements, working up to a full description of the theory of generation of expressions. This is open ended, and there's lots of interesting possibility.