Math 2001: Setting up Proof by Contrapositive (Katherine Stange, Spring 2018)

For each theorem, set up the form of a proof by contrapositive. Do *not* write a full proof. That is, write the first sentence (or two) of the proof (that is, the assumptions), and the last sentence of the proof (that is, the conclusion). Do so in such a way as to demonstrate the structure of proof by contrapositive.

Theorem 1 (Example Theorem)**.** *Let x be a real number. If x is irrational, then x has a non-periodic decimal expansion.*

1. First sentence(s): Suppose that x is a real number with a periodic decimal expansion.
2. Last sentence(s): Therefore x is rational.

Theorem 2. *Let n be an integer. If n^2 is a power of two, then n is a power of two.*

1. First sentence(s): Suppose n is an integer which is not a power of two.
2. Last sentence(s): We have shown n^2 is not a power of two.

Note: Solutions vary! Yours does not need to be word for word the same.

Theorem 3. *For any integer n such that n^2 is odd, it must be that n is odd.*

1. First sentence(s): Suppose n is an even integer.
2. Last sentence(s): Thus n^2 is even.

Theorem 4. *Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. If the derivative f' is identically zero, then f is a constant function.*

1. First sentence(s): Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Suppose that f is not a constant function.
2. Last sentence(s): Therefore f' is not identically zero.

Theorem 5. *Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function whose derivative is identically zero. Then f is a constant function.*

Note: This is a rephrasing of the previous theorem. The same solution works.

Theorem 6. *Let n and m be negative integers. Then nm is a positive integer.*

To help set up a contrapositive proof, here's a rephrasing of the theorem:

Theorem 7. *Let $n, m \in \mathbb{Z}$. If n and m are negative, then nm is a positive integer.*

1. First sentence(s): Let $n, m \in \mathbb{Z}$. Suppose also that nm is not a positive integer.
2. Last sentence(s): Therefore n and m are not both negative.

Note: There are other ways you could rephrase the theorem as an *if-then*, and give correct contrapositive proofs. Here is an example (albeit a quirky one…this probably isn't the best way to organize a proof!):

Theorem 8. *Let n and m be negative real numbers. If n and m are integers, then nm is a positive integer.*

1. First sentence(s): Let $n, m \in \mathbb{R}$, and suppose $n, m < 0$. Suppose also that $nm \notin \mathbb{N}$.
2. Last sentence(s): Therefore n and m are not both integers.