## Math 2001: Setting up Proof by Contradiction (Katherine Stange, Spring 2019)

For each theorem, set up the form of a proof by contradiction. Do *not* write a full proof. That is, write the first sentence (or two) of the proof (that is, the assumptions), and the last sentence of the proof (that is, the conclusion). Do so in such a way as to demonstrate the structure of proof by contradiction. Please simplify the negation.

**Theorem 1** (Example Theorem)**.** *Let $x$ be a positive real number. Then all real numbers larger than $x$ are positive.*

1. First sentence(s): Let $x$ be a positive real number. Suppose, for a contradiction, that it is not true that all real numbers larger than $x$ are positive. In other words, there is some real number $y$ which is larger than $x$ and which is non-positive.
2. Last sentence(s): We have reached a contradiction, proving the theorem.

**Theorem 2.** *The square root of 2 is irrational.*

1. First sentence(s):
2. Last sentence(s):

**Theorem 3.** *Let $n$ be an integer. Suppose $n^2$ is odd. Then $n$ is odd.*

1. First sentence(s):
2. Last sentence(s):

**Theorem 4.** *Let $n$ and $m$ be negative integers. Then $nm$ is a positive integer.*

1. First sentence(s):
2. Last sentence(s):

**Theorem 5.** *Suppose $a$ and $b$ are positive integers. Then $a^2 + b^2$ is positive.*

1. First sentence(s):
2. Last sentence(s):

**Theorem 6.** *If $a$ and $b$ are positive integers, then $a^2 + b^2$ is positive.*

1. First sentence(s):
2. Last sentence(s):

**Theorem 7.** *Let $f : \mathbb{R} \to \mathbb{R}$ be a function whose derivative is identically zero. Then $f$ is a constant function.*

1. First sentence(s):

2. Last sentence(s):

**Theorem 8.** *Let $f : \mathbb{R} \to \mathbb{R}$ be a function. If the derivative $f'$ is identically zero, then $f$ is a constant function.*

1. First sentence(s):

2. Last sentence(s):

**Theorem 9.** *There is no real number $x$ such that $x^2 < -x^2$.*

1. First sentence(s):

2. Last sentence(s):

**Theorem 10.** *There do not exist any positive integers which square to zero.*

1. First sentence(s):

2. Last sentence(s):

**Theorem 11.** *All perfect squares are non-negative.*

1. First sentence(s):

2. Last sentence(s):

**Theorem 12.** *All real numbers $x$ satisfy $x^2 \geq 0$.*

1. First sentence(s):

2. Last sentence(s):